What does PISA really assess? What it doesn't? A French view Part 2 - released math questions with some results

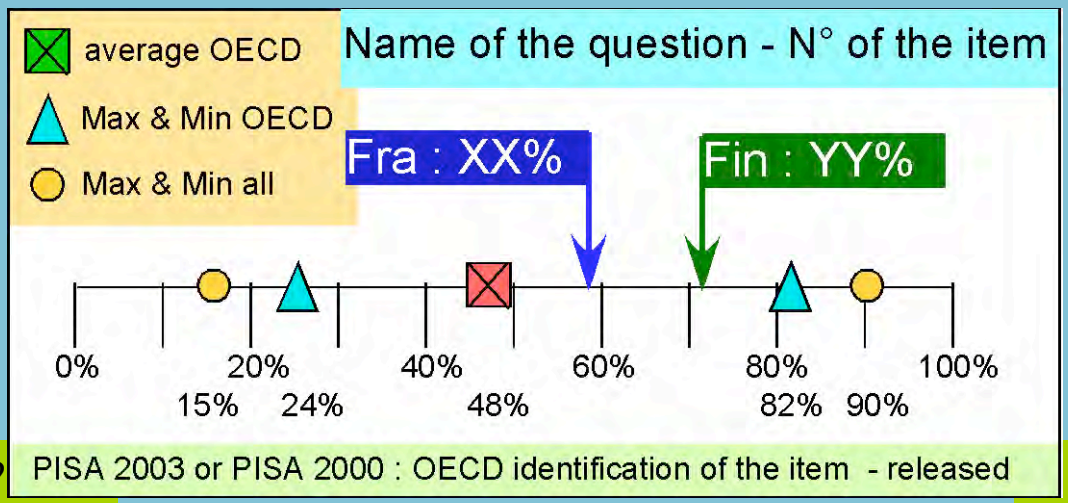
Joint Finnish-French Conference
"L'enseignement des mathématiques : à partir de l'enquête PISA"
"Teaching mathematics: beyond the PISA survey »
Paris 6 - 8 octobre 2005

Antoine Bodin
Institut de Recherche sur l'Enseignement des Mathématiques
Université of Franche-Comté

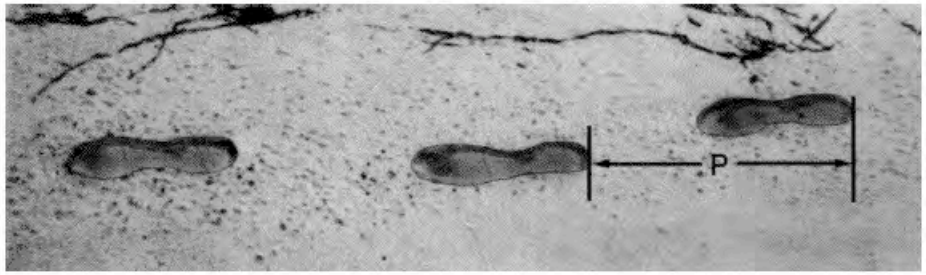
The following slides display all released mathematics questions 2000 and 2003

With average percentage results obtained for each particular item:

- In the whole OECD,
- In France
- In Finland
- By the less and most successful OECD countries
- By the less and most successful participating countries



WALKING



The picture shows the footprints of a man walking. The pacelength *P* is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between *n* and *P* where,

n = number of steps per minute, and

P = pacelength in metres.

Question 1: WALKING *M124Q01- 0 1 2 9*

If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

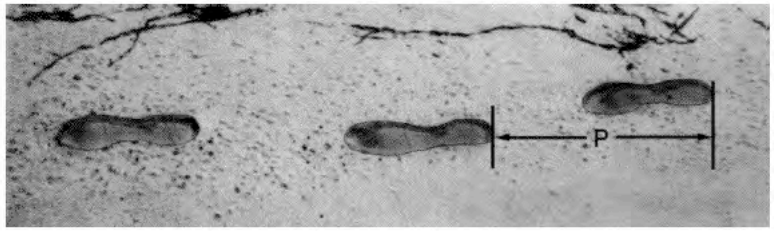
Question 3: WALKING *M124Q03- 00 11 21 22 23 24 31 99*

Bernard knows his pacelength is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

M124_ WALKING 2

WALKING



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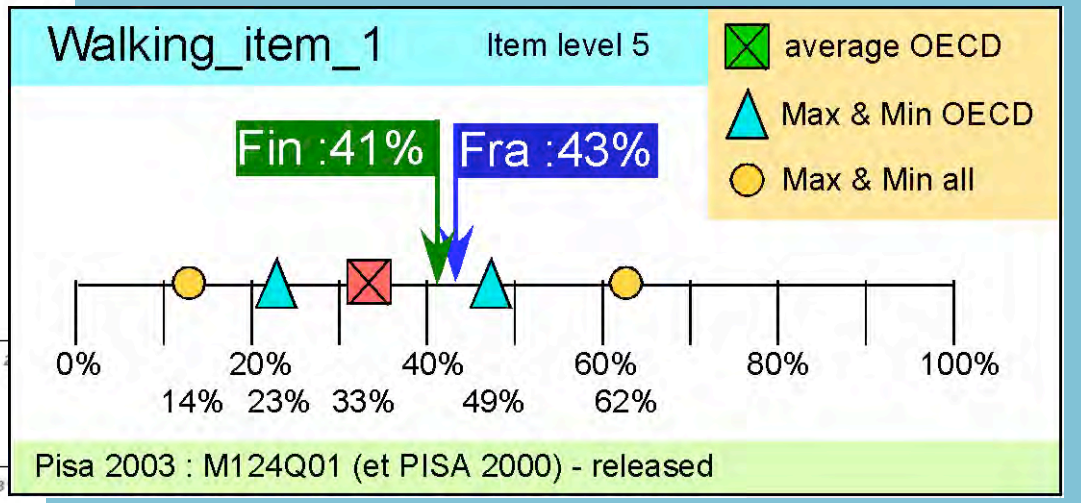
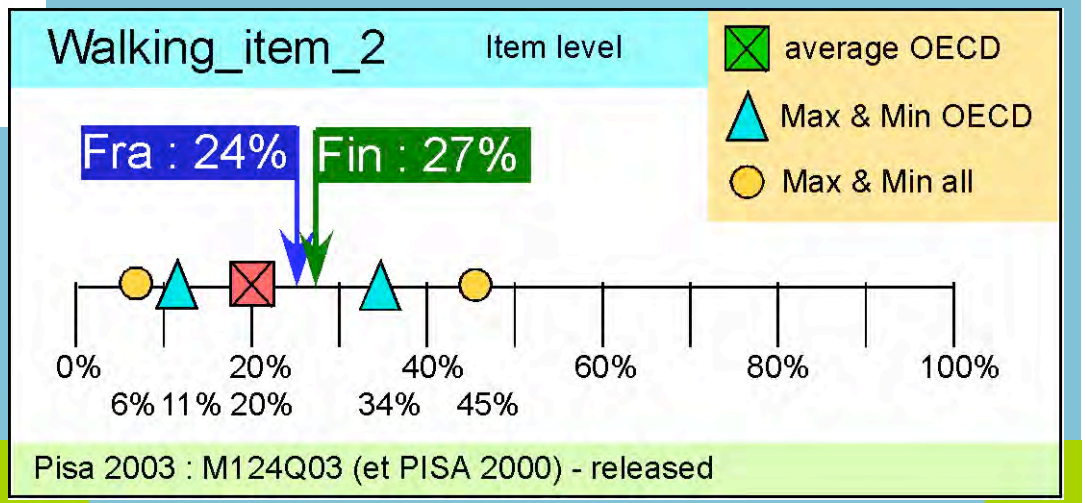


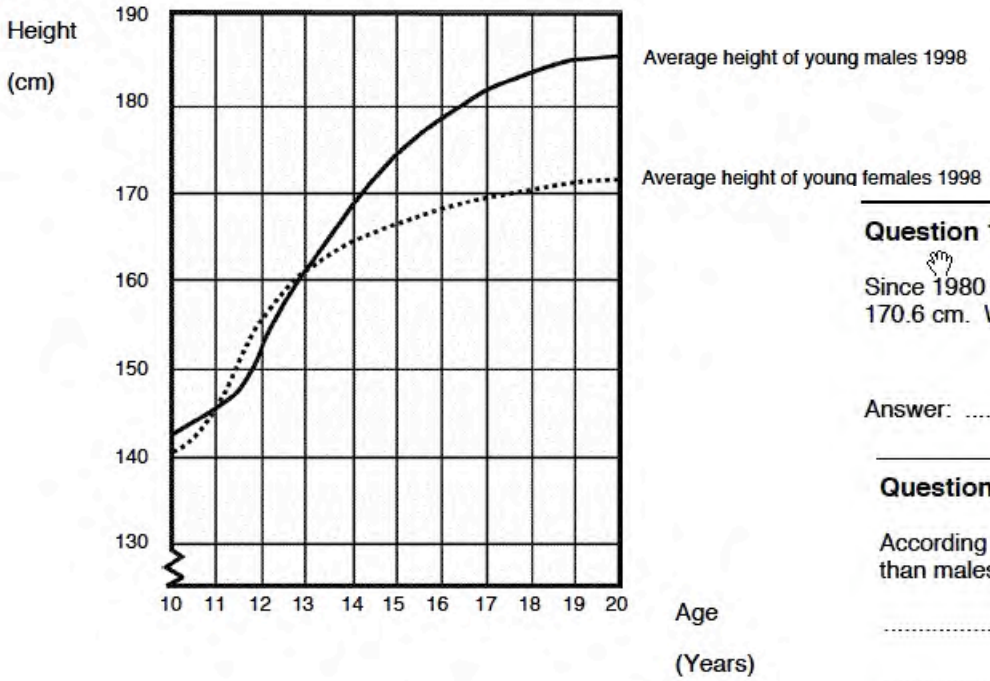


M150_GROWING UP 1

GROWING UP

YOUTH GROWS TALLER

In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.



Question 1: GROWING UP

M150Q01- 0 1 9

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

Answer: .....cm

Question 2: GROWING UP

M150Q02- 00 11 21 22 99

According to this graph, on average, during which period in their life are females taller than males of the same age?

.....

.....

Question 3: GROWING UP

M150Q03- 01 02 11 12 13 99

Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.

.....

.....

.....

What does PISA really assess.. in maths?

M150_GROWING UP 2

GROWING UP

YOUTH GROWS TALLER

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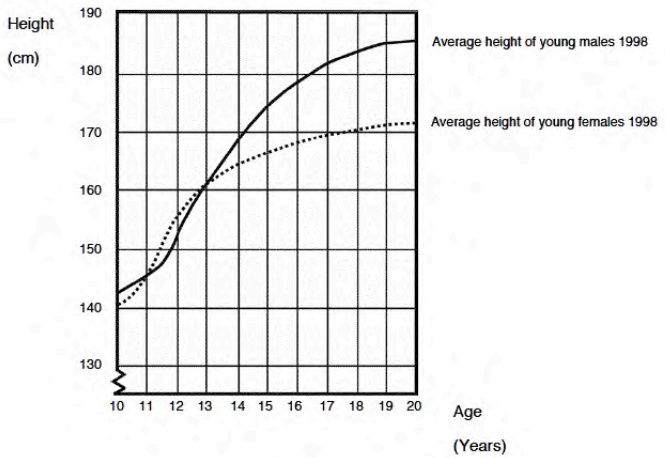

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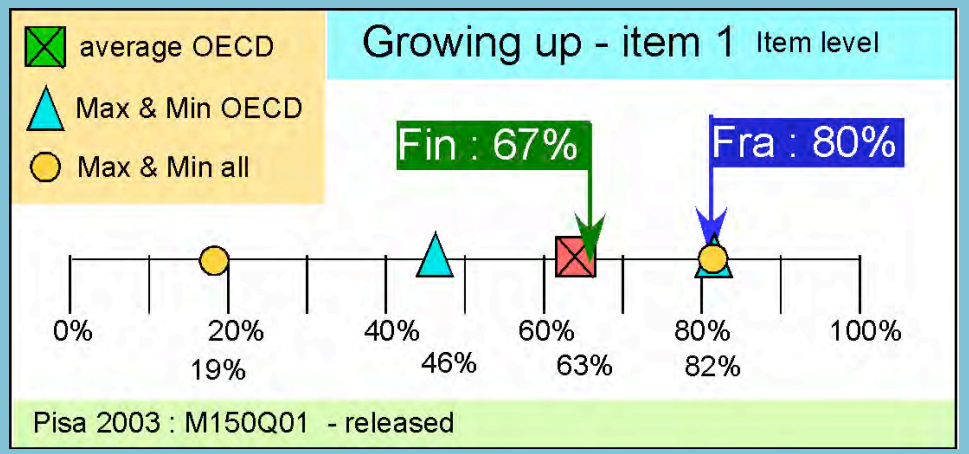
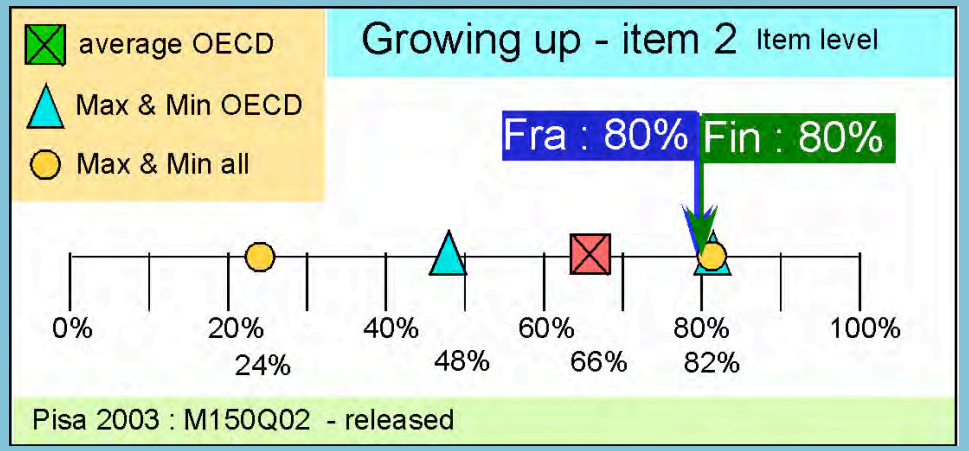
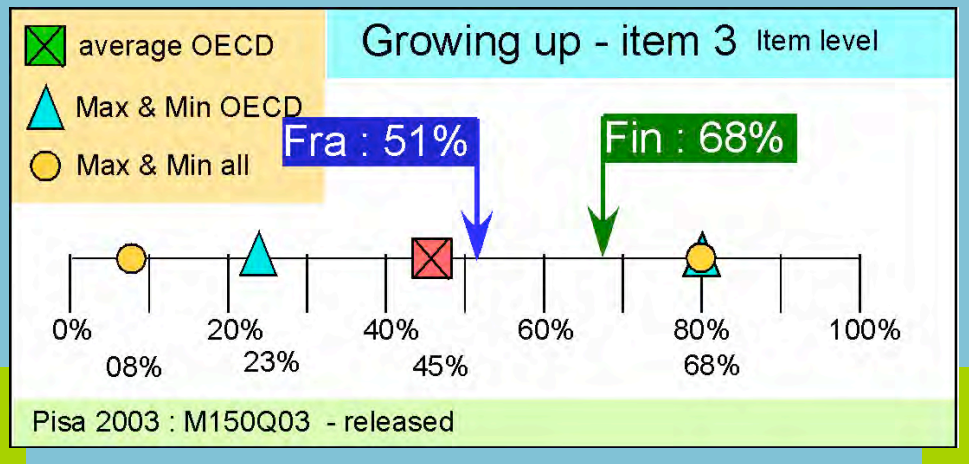




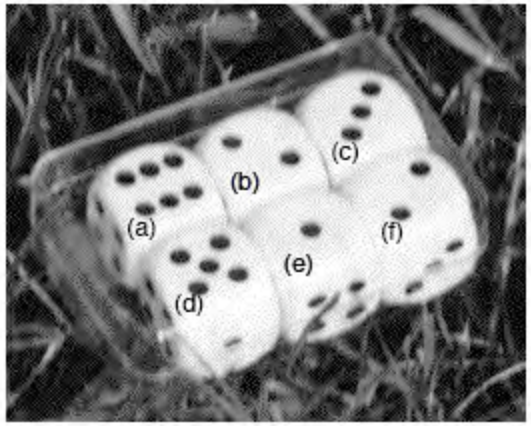


CUBES

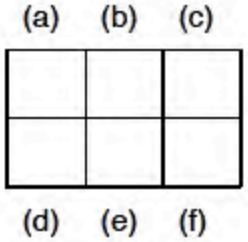
Question 1: CUBES *M145Q01*

In this photograph you see six dice, labelled (a) to (f). For all dice there is a rule:

The total number of dots on two opposite faces of each die is always seven.



Write in each box the number of dots on the **bottom** face of the dice corresponding to the photograph.



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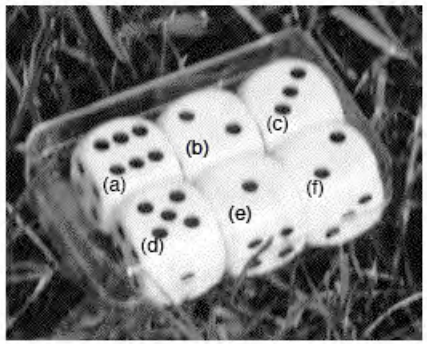
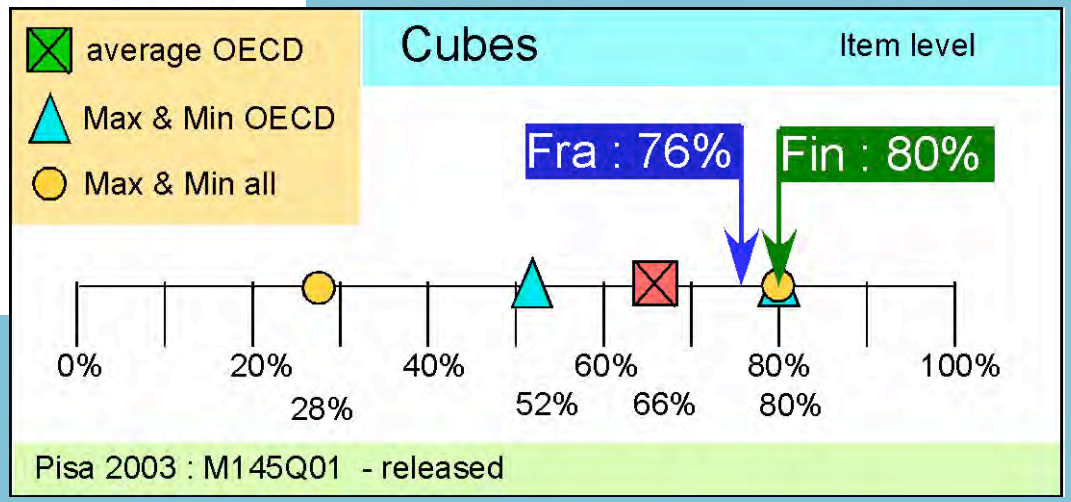


Write in each box the number of dots on the **bottom** face of the dice corresponding to the photograph.

(a) (b) (c)

(d) (e) (f)



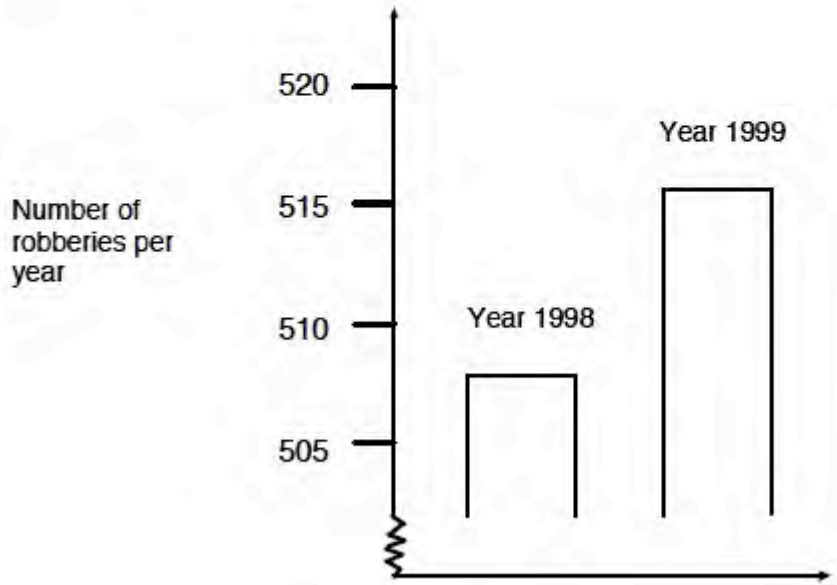
ROBBERIES

Question 1: ROBBERIES

M179Q01- 01 02 03 04 11 12 21 22 23 99

A TV reporter showed this graph and said:

"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

M179_ROBBERIES 2 + EVAPM

ROBBERIES

Question 1: ROBBERIES *M179Q01- 01 02 03 04 11 12 21 22 23 99*

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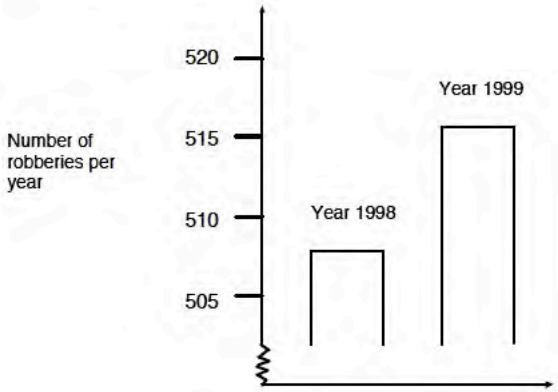

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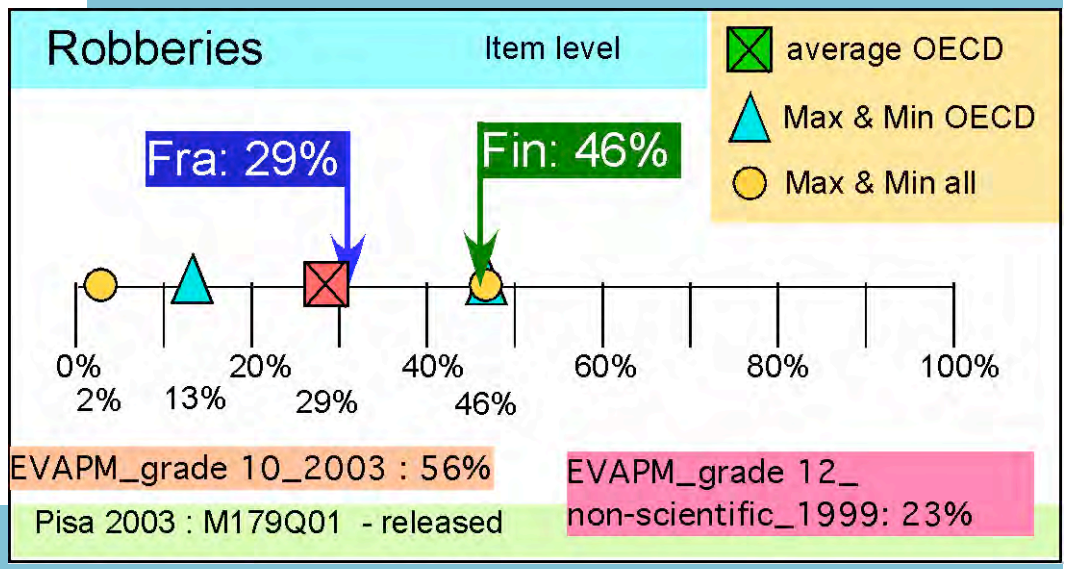
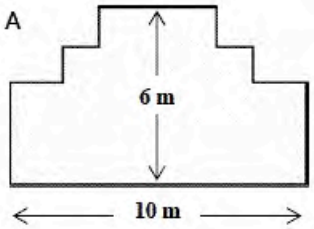


CARPENTER

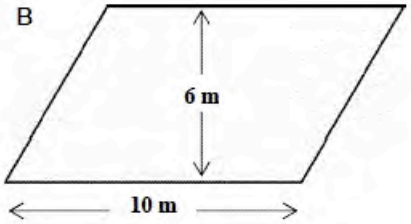
Question 1: CARPENTER M266Q01

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.

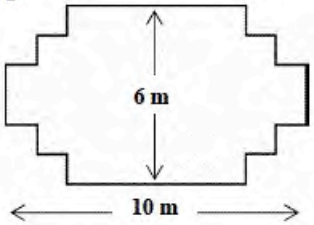
A



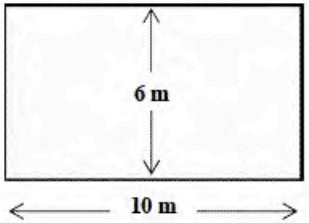
B



C



D



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

M266_CARPENTER 2

CARPENTER

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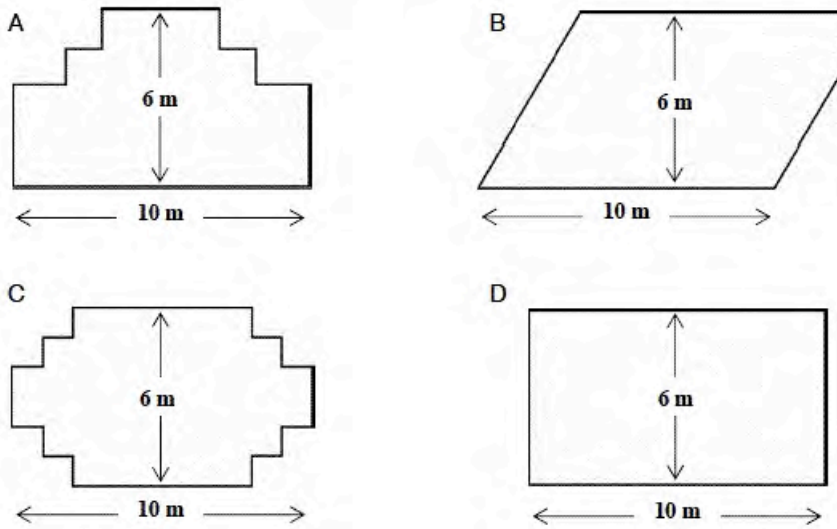

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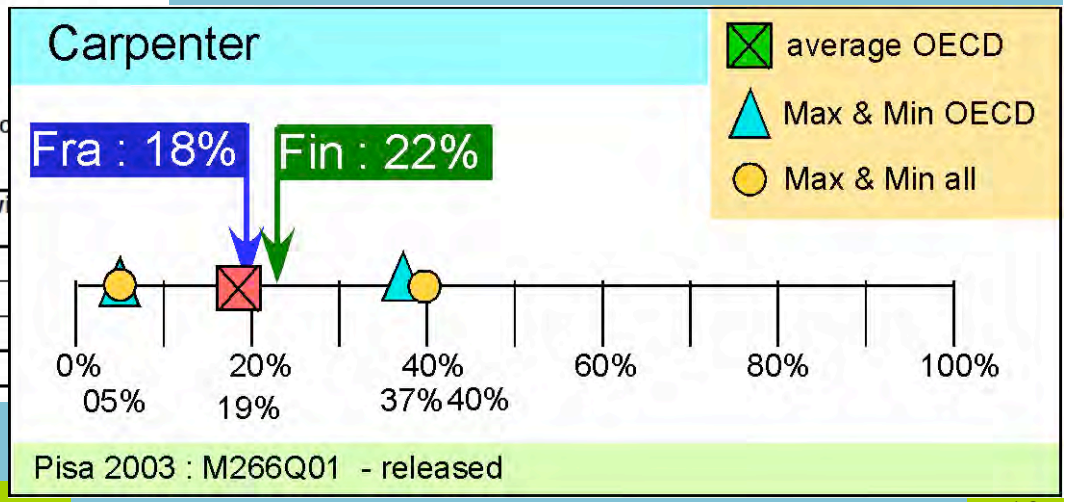

M402_INTERNET RELAY CHAT 1

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using "chat" on the Internet. They have to log on to the Internet at the same time to be able to chat.

To find a suitable time to chat, Mark looked up a chart of world times and found the following:

Question 1: INTERNET RELAY CHAT *M402Q01 - 0 1 9*

At 7:00 PM in Sydney, what time is it in Berlin?

Answer: .....

Question 2: INTERNET RELAY CHAT *M402Q02 - 0 1 9*

Mark and Hans are not able to chat between 9:00 AM and 4:30 PM their local time, as they have to go to school. Also, from 11:00 PM till 7:00 AM their local time they won't be able to chat because they will be sleeping.

When would be a good time for Mark and Hans to chat? Write the local times in the table.

Place	Time
Sydney	
Berlin	

M402_INTERNET RELAY CHAT 2

INTERNET RELAY CHAT

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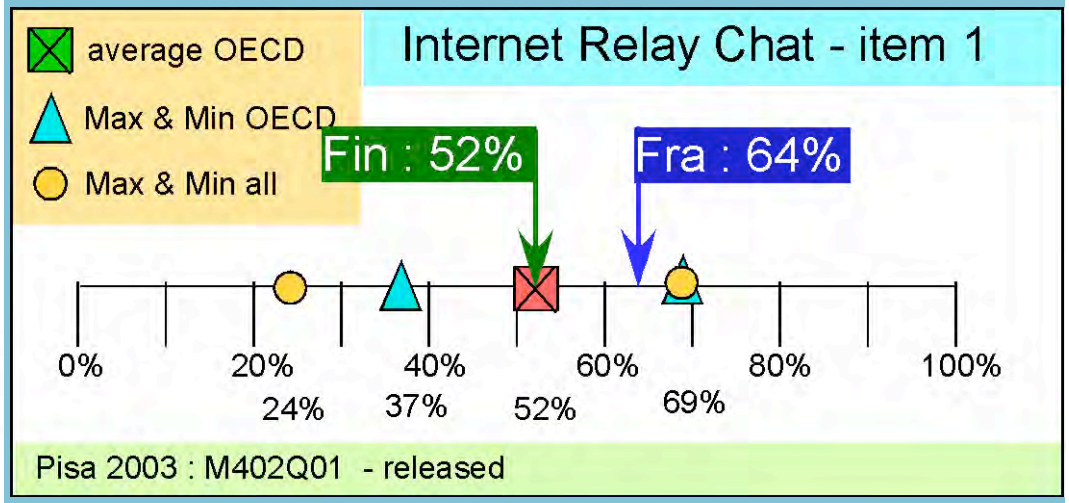

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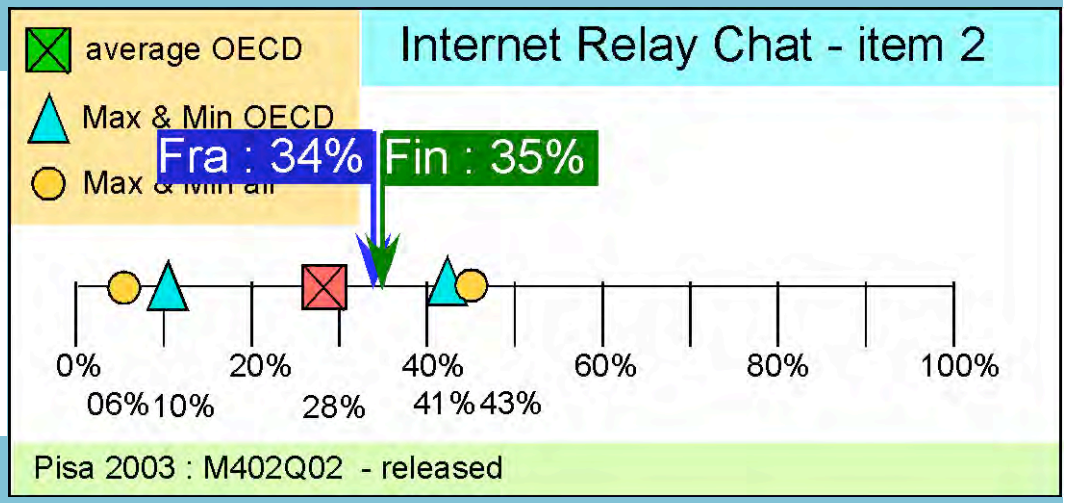
Berlin	



EXCHANGE RATE

Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

Question 1: EXCHANGE RATE

M413Q01 - 0 1 9

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was:

1 SGD = 4.2 ZAR

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

Answer: .....

Question 2: EXCHANGE RATE

M413Q02 - 0 1 9

On returning to Singapore after 3 months, Mei-Ling had 3 900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:

1 SGD = 4.0 ZAR

How much money in Singapore dollars did Mei-Ling get?

Answer: .....

Question 3: EXCHANGE RATE

M413Q03 - 01 02 11 99

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

M413_EXCHANGE RATE 2

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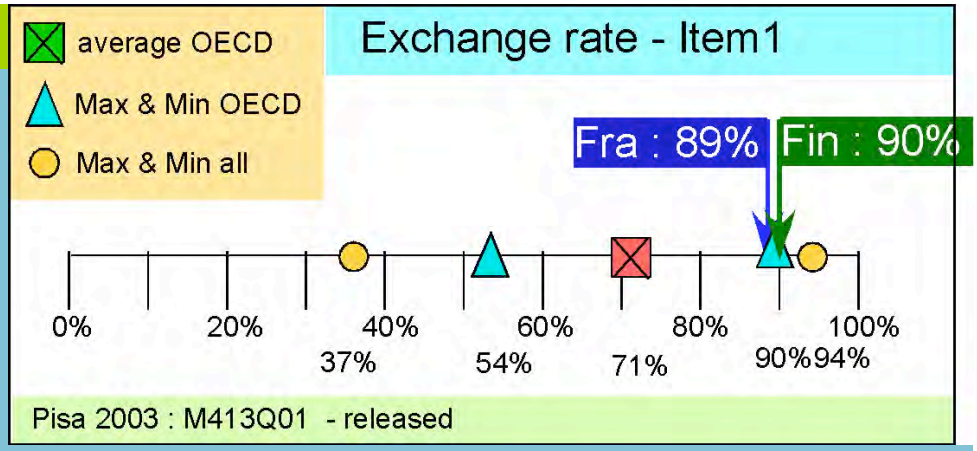
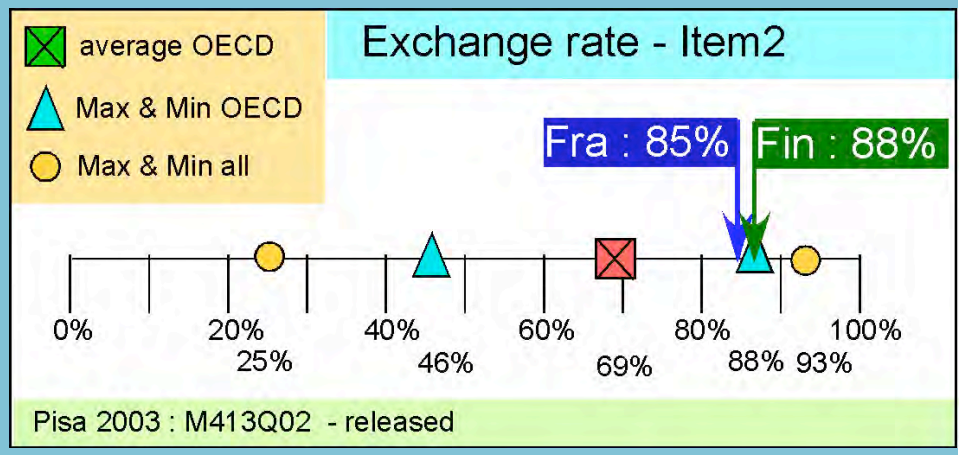
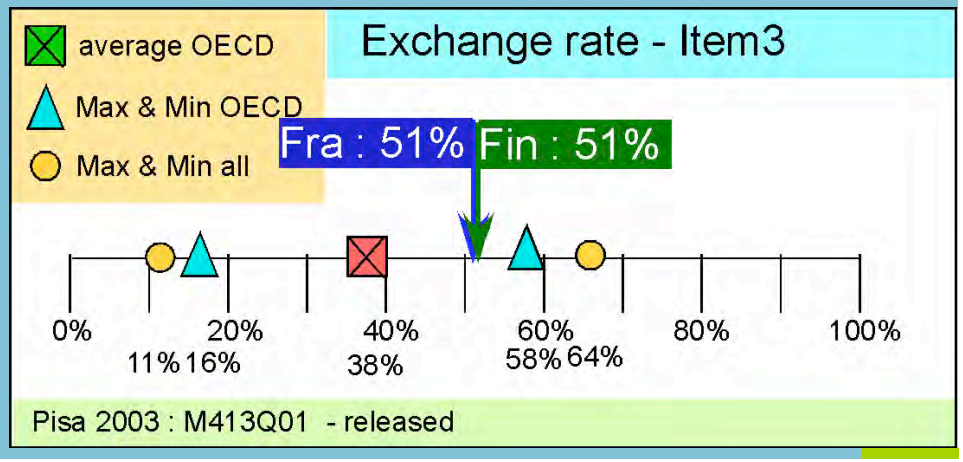






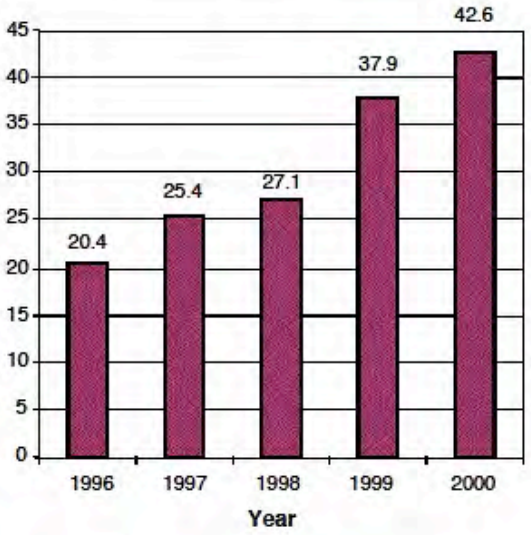
What does PISA really assess.. in maths?

M438_EXPORTS 1

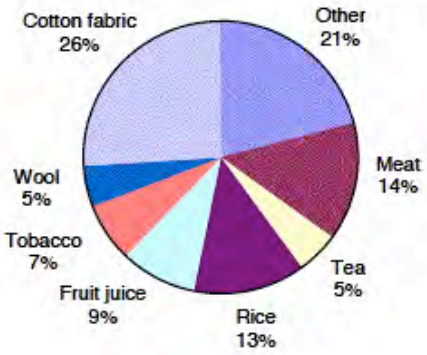
EXPORTS

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.

Total annual exports from Zedland in millions of zeds, 1996-2000



Distribution of exports from Zedland in 2000



Question 1: EXPORTS

M438Q01 - 0 1 9

What was the total value (in millions of zeds) of exports from Zedland in 1998?

Answer: .....

Question 2: EXPORTS

M438Q02

What was the value of fruit juice exported from Zedland in 2000?

A 1.8 million zeds.
B 2.3 million zeds.
C 2.4 million zeds.
D 3.4 million zeds.
E 3.8 million zeds.

What does PISA really assess.. in mat

M438_EXPORTS 2

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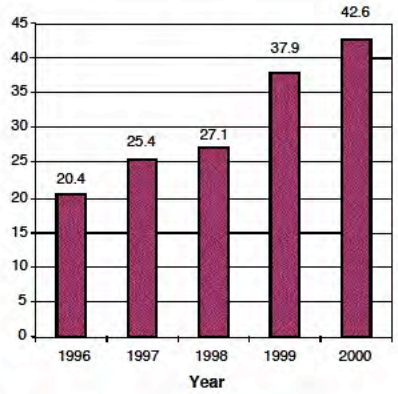

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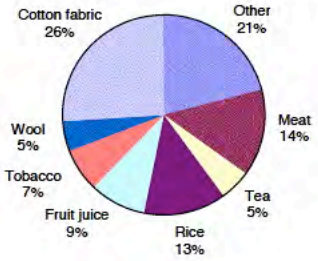

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M438Q01 - 0 1 9

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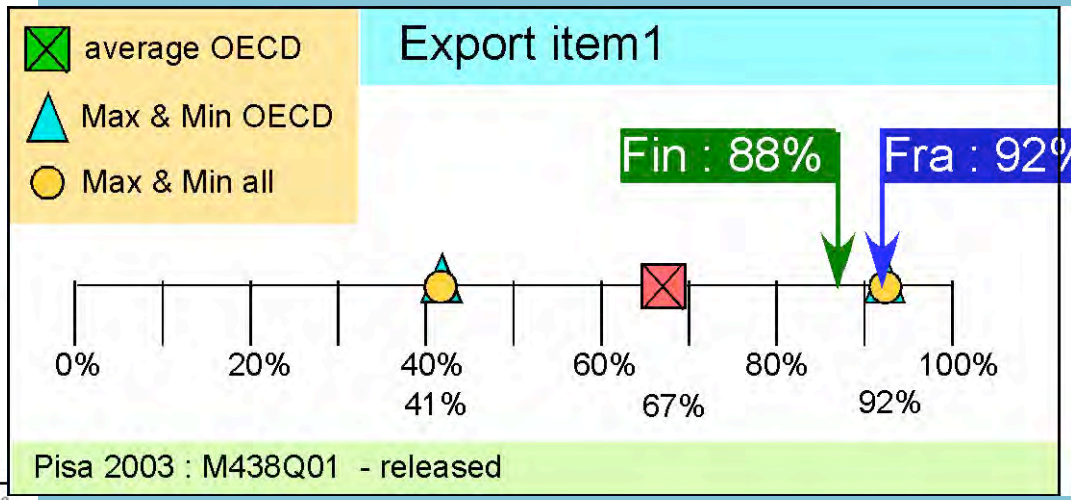

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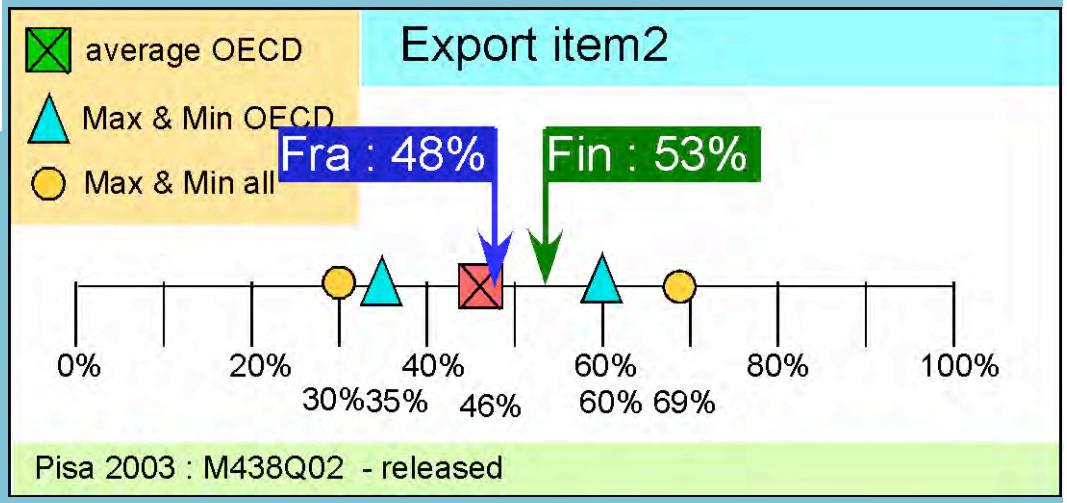
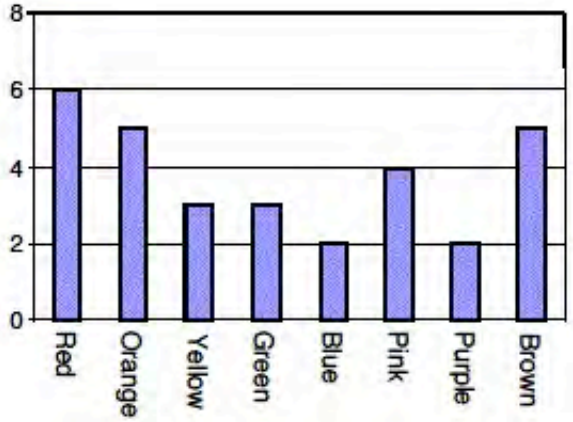
E 3.8 million zeds.



COLOURED CANDIES

Question 1: COLOURED CANDIES M467Q01

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each colour in the bag is shown in the following graph.



What is the probability that Robert will pick a red candy?

A 10%
B 20%
C 25%
D 50%

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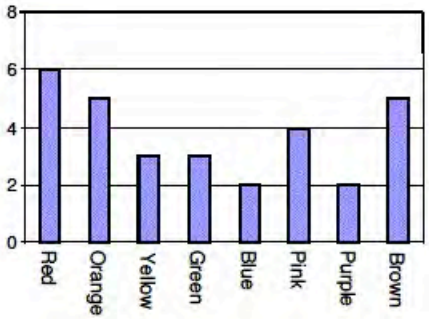

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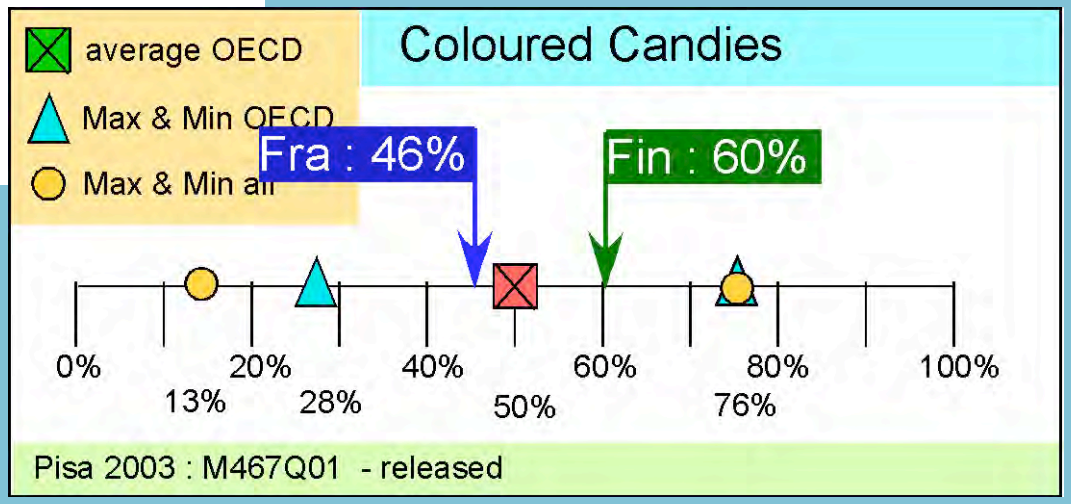
C 25%
D 50%



SCIENCE TESTS

Question 1: SCIENCE TESTS

M468Q01

In Mei Lin's school, her science teacher gives tests that are marked out of 100. Mei Lin has an average of 60 marks on her first four Science tests. On the fifth test she got 80 marks.

What is the average of Mei Lin's marks in Science after all five tests?

Average: .....

SCIENCE TESTS

Question 1: SCIENCE TESTS *M468Q01*

In Mei Lin's school, her science teacher gives tests that are marked out of 100. Mei Lin has an average of 60 marks on her first four Science tests. On the fifth test she got 80 marks.

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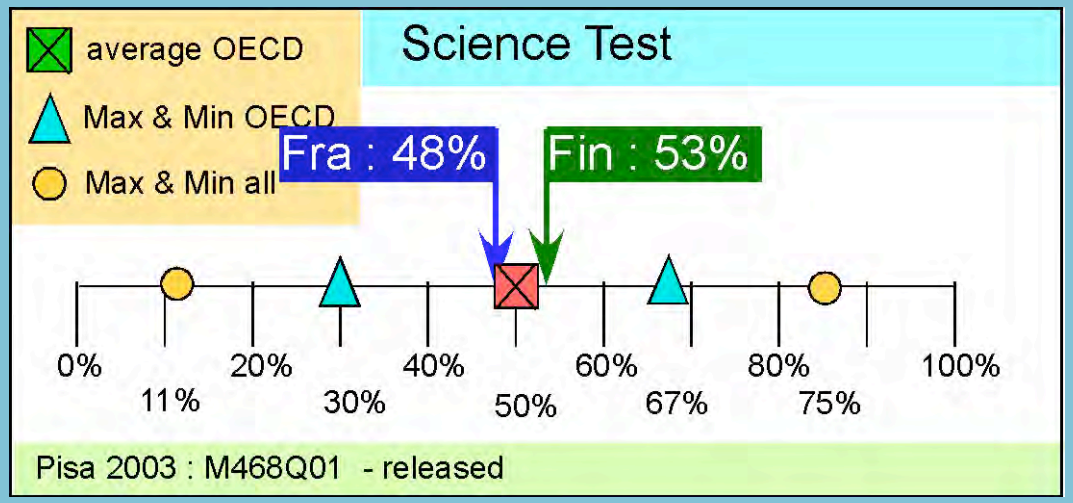

BOOKSHELVES

Question 1: BOOKSHELVES

M484Q01

To complete one set of bookshelves a carpenter needs the following components:

4 long wooden panels,

6 short wooden panels,

12 small clips,

2 large clips and

14 screws.

The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

Answer: .....

BOOKSHELVES

Question 1: BOOKSHELVES M484Q01

To complete one set of bookshelves a carpenter needs the following components:

4 long wooden panels,

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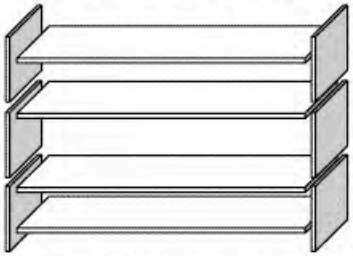

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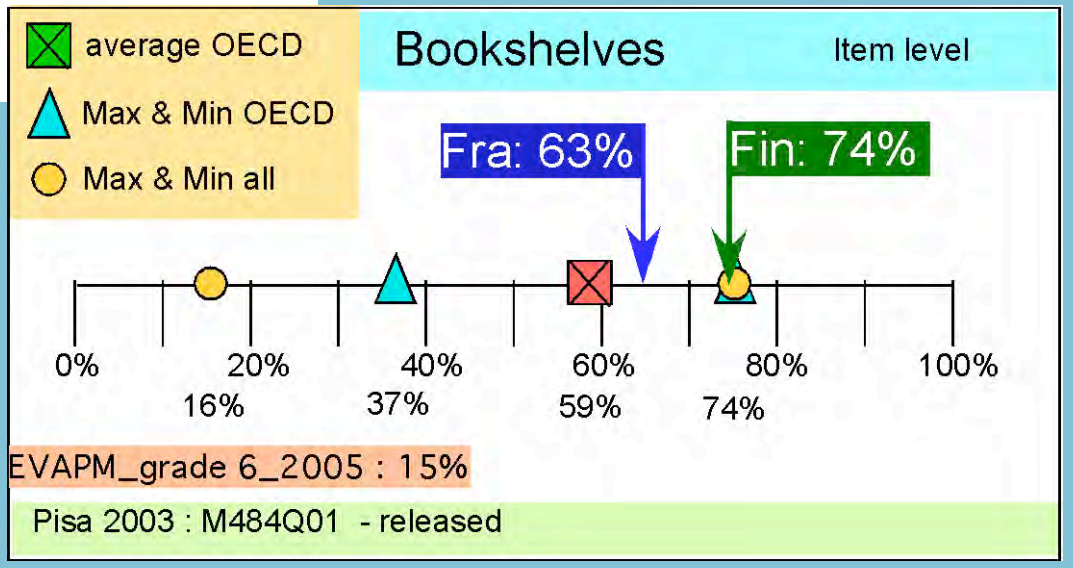
Answer: .....



LITTER

Question 1: LITTER

M505Q01 - 0 1 9

For a homework assignment on the environment, students collected information on the decomposition time of several types of litter that people throw away:

Type of Litter	Decomposition time
Banana peel	1–3 years
Orange peel	1–3 years
Cardboard boxes	0.5 year
Chewing gum	20–25 years
Newspapers	A few days
Polystyrene cups	Over 100 years

A student thinks of displaying the results in a bar graph.

Give **one** reason why a bar graph is unsuitable for displaying these data.

LITTER

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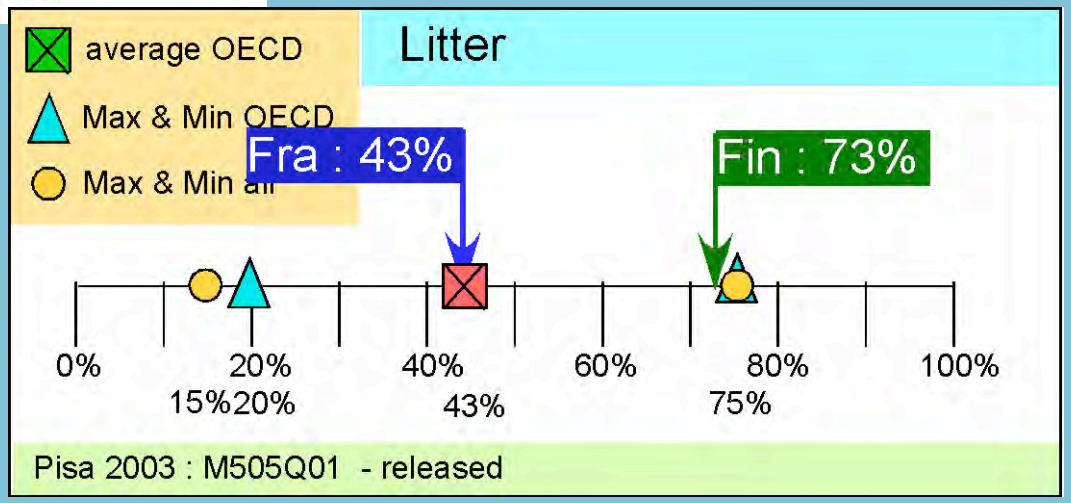

EARTHQUAKE

Question 1: EARTHQUAKE *M509Q01*

A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes.

A geologist stated: "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three".

Which of the following best reflects the meaning *of the geologist's statement*?

A $\frac{2}{3} \times 20 = 13.3$, so between 13 and 14 years from now there will be an earthquake in Zed City.

B $\frac{2}{3}$ is more than $\frac{1}{2}$, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.

C The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.

D You cannot tell what will happen, because nobody can be sure when an earthquake will occur.

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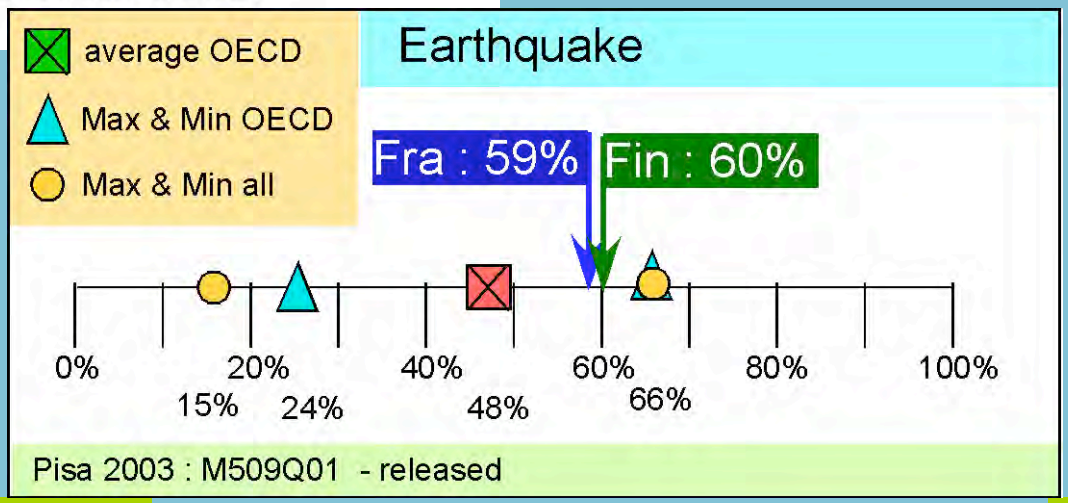

CHOICES

Question 1: CHOICES *M510Q01*

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami.

Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

Answer: ..... combinations.

CHOICES

Question 1: CHOICES *M510Q01*

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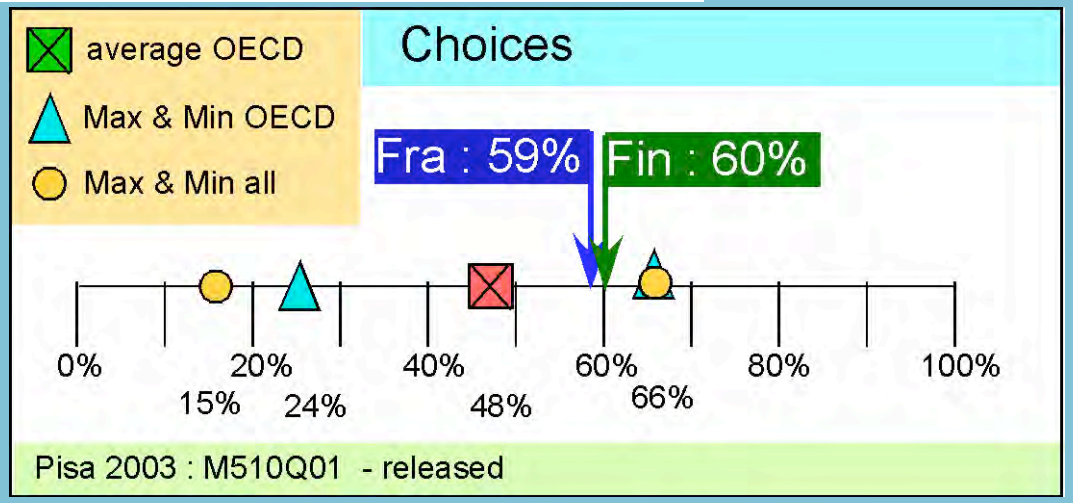


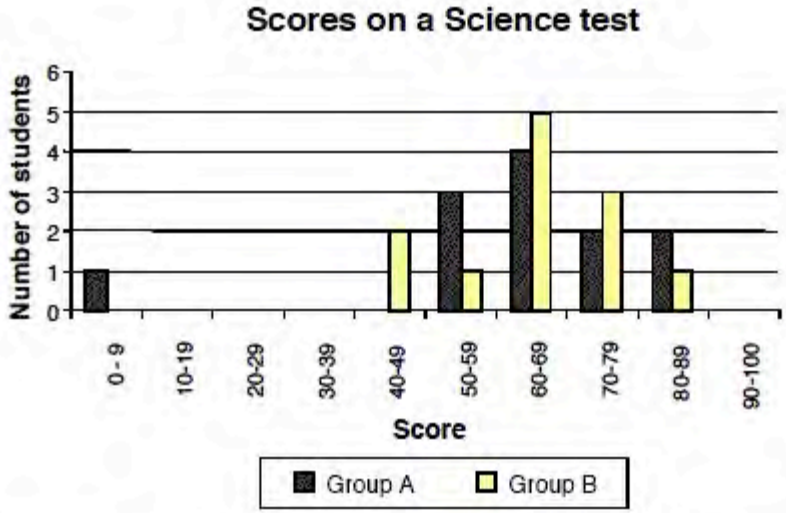
TEST SCORES

Question 1: TEST SCORES

M513Q01 - 0 1 9

The diagram below shows the results on a Science test for two groups, labelled as Group A and Group B.

The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.



Looking at the diagram, the teacher claims that Group B did better than Group A in this test.

The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better.

Give one mathematical argument, using the graph, that the students in Group A could use.

Delete teacher necessa

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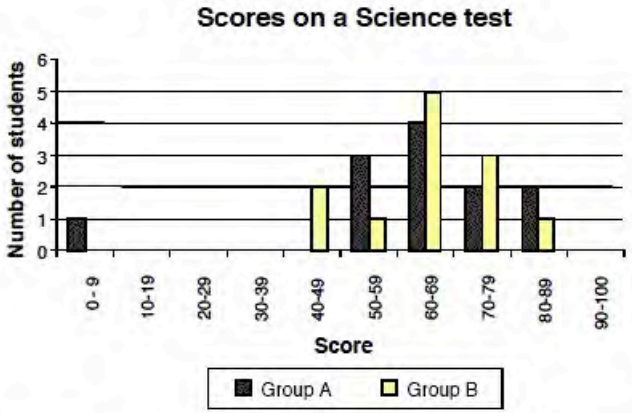

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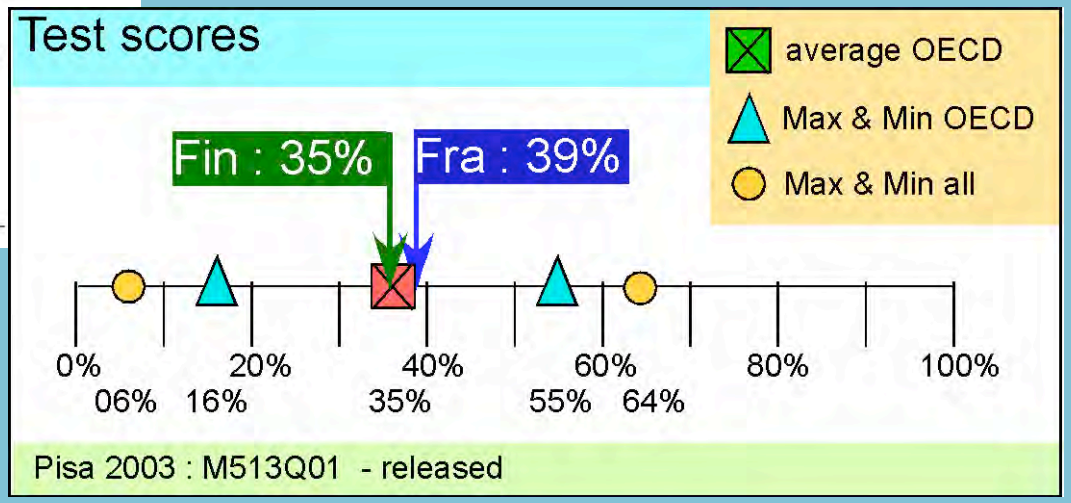

M520_SKATEBOARD 1

SKATEBOARD

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Question 1: SKATEBOARD

M520Q01a

M520Q01b

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: ..... zeds.

(b) Maximum price: ..... zeds.

Question 2: SKATEBOARD

M520Q02

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

A 6
B 8
C 10
D 12

Question 3: SKATEBOARD

M520Q03

Eric has 120 zeds to spend and wants to buy the most expensive skateboard he can afford.

How much money can Eric afford to spend on each of the 4 parts? Put your answer in the table below.

Part	Amount (zeds)
Deck	
Wheels	
Trucks	
Hardware	

What does PISA really assess.. in maths?

M520_SKATEBOARD 2

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
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One set of 4 Wheels	14 or 36	
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One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Question 1: SKATEBOARD M520Q01a M520Q01b

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: ..... zeds.

(b) Maximum price: ..... zeds.

Question 2: SKATEBOARD M520Q02

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

A 6
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Question 3: SKATEBOARD M520Q03

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Part	Amount (zeds)
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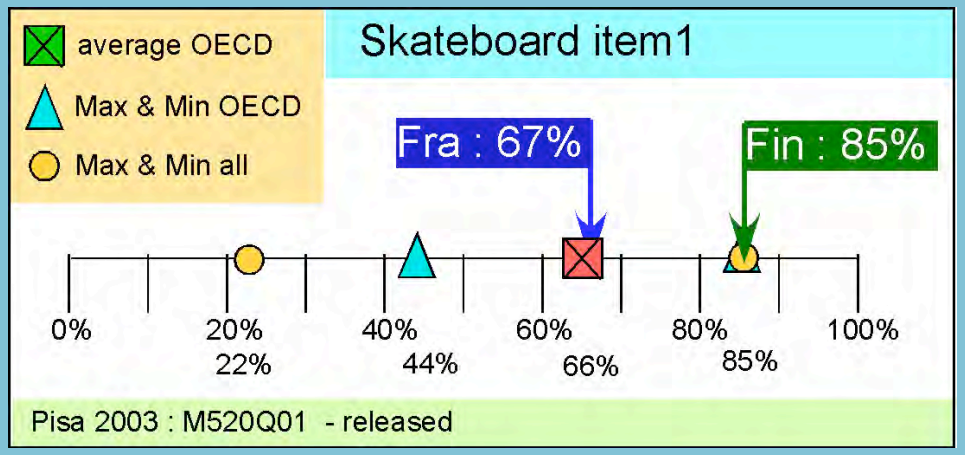
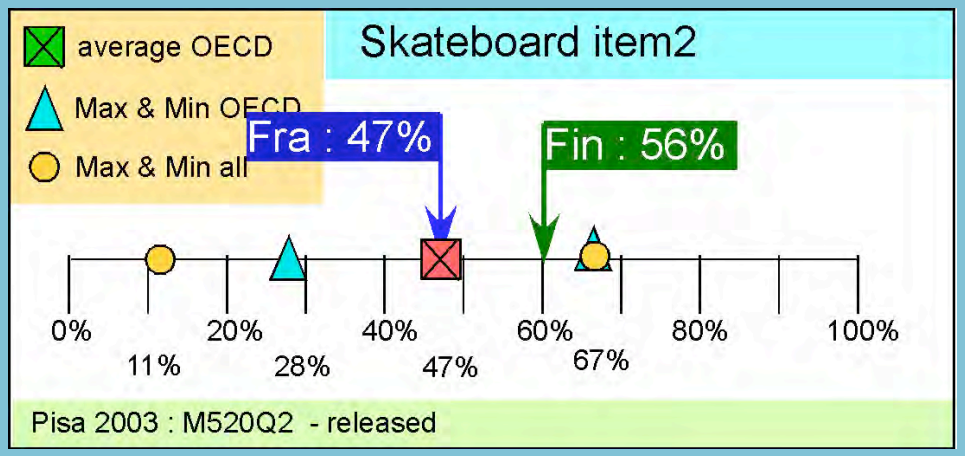
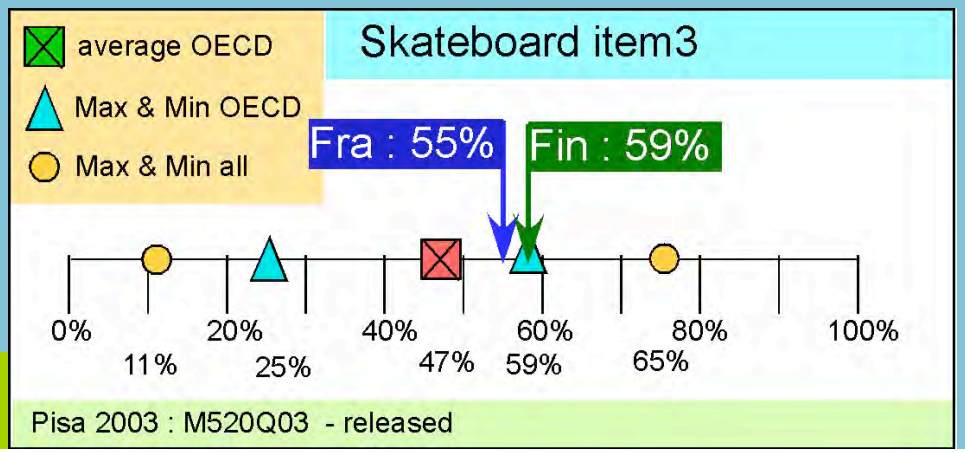
Hardware	







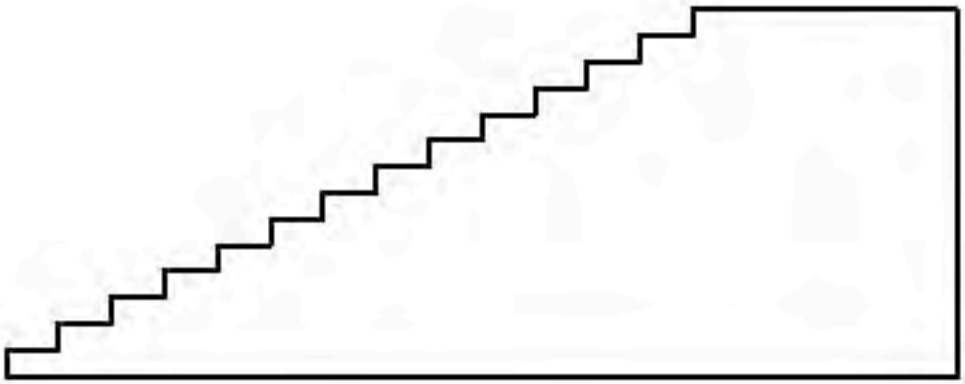
ths?

STAIRCASE

Question 1: STAIRCASE

M547Q01

The diagram below illustrates a staircase with 14 steps and a total height of 252 cm:



What is the height of each of the 14 steps?

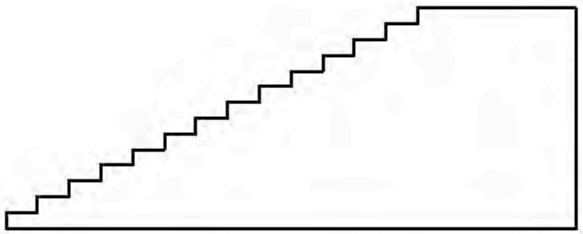
Height: ..... cm.

M547_STAIRCASE 2

STAIRCASE

Question 1: STAIRCASE M547Q01

The diagram below illustrates a staircase with 14 steps and a total height of 252 cm:

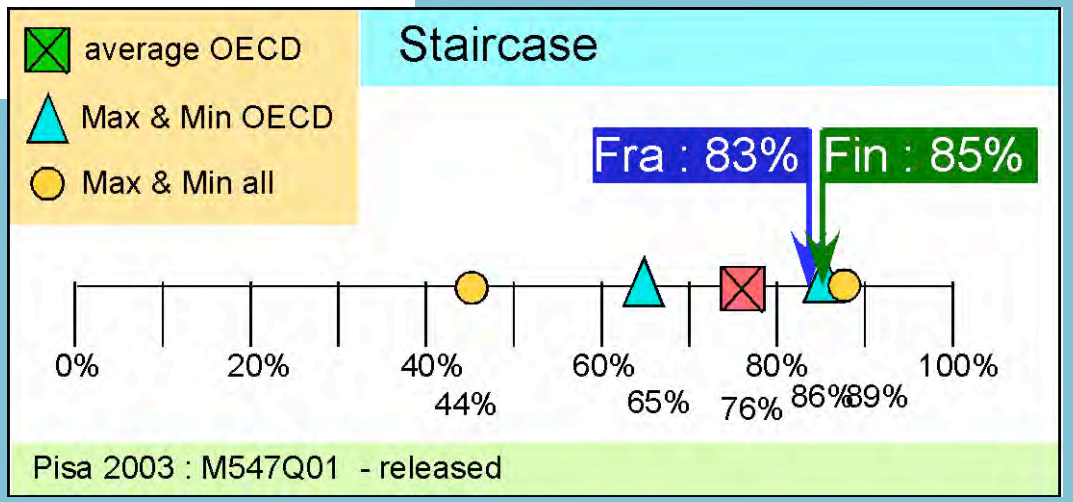


Total height 252 cm

Total depth 400 cm

What is the height of each of the 14 steps?

Height: ..... cm.



Question 2: NUMBER CUBES

M555Q02

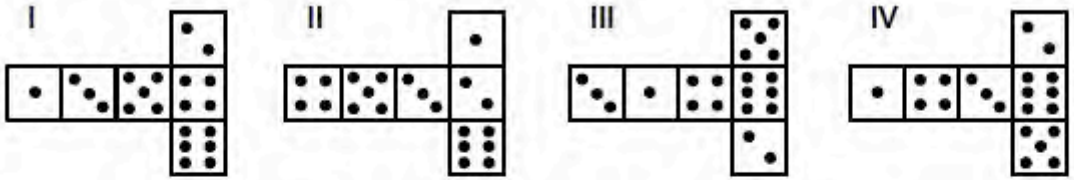
On the right, there is a picture of two dice.

Dice are special number cubes for which the following rule applies:

> The total number of dots on two opposite faces is always seven.

You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways. In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either "Yes" or "No" in the table below.



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

NUMBER CUBES

Question 2: NUMBER CUBES M555Q02

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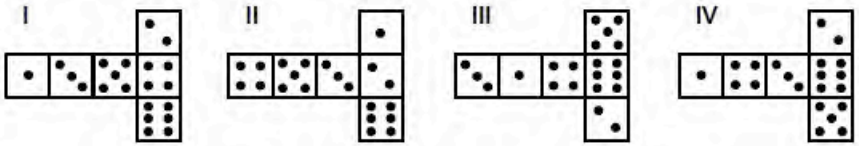

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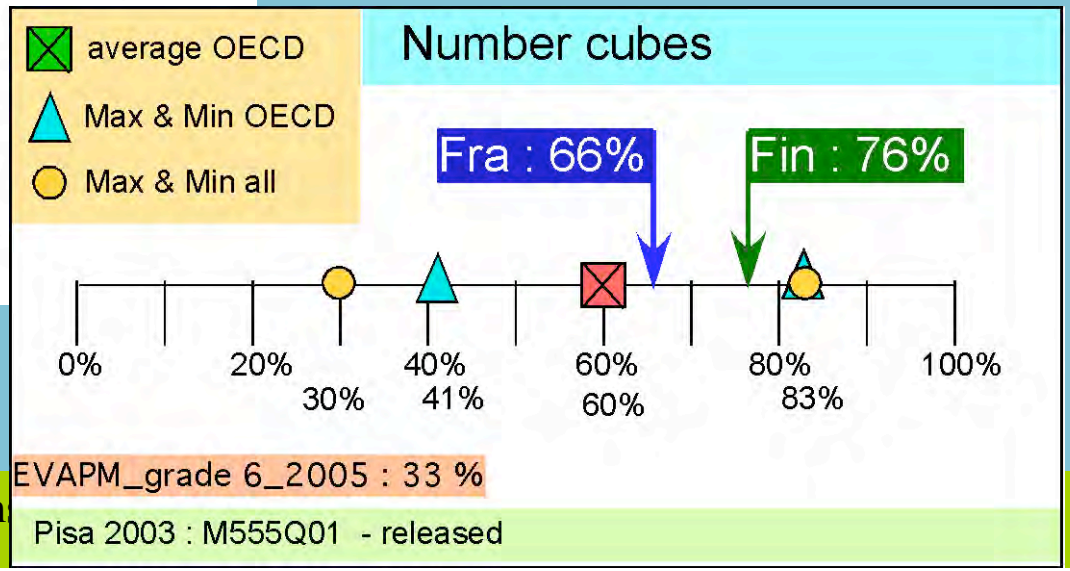

SUPPORT FOR THE PRESIDENT

Question 1: SUPPORT FOR THE PRESIDENT

M702Q01 - 0 1 2 9

In Zedland, opinion polls were conducted to find out the level of support for the President in the forthcoming election. Four newspaper publishers did separate nationwide polls. The results for the four newspaper polls are shown below:

Newspaper 1: 36.5% (poll conducted on January 6, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 2: 41.0% (poll conducted on January 20, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 3: 39.0% (poll conducted on January 20, with a sample of 1000 randomly selected citizens with voting rights)

Newspaper 4: 44.5% (poll conducted on January 20, with 1000 readers phoning in to vote).

Which newspaper's result is likely to be the best for predicting the level of support for the President if the election is held on January 25? Give two reasons to support your answer.

M702_SUPPORT FOR THE PRESIDENT 2

SUPPORT FOR THE PRESIDENT

Question 1: SUPPORT FOR THE PRESIDENT *M702Q01 - 0 1 2 9*

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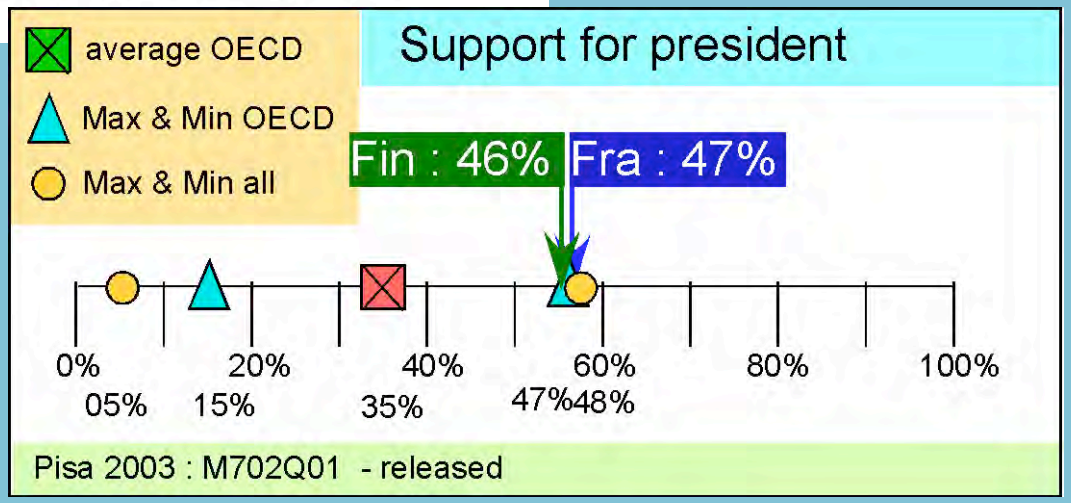
Newspaper 4: 44.5% (poll conducted on January 20, with 1000 readers phoning in to vote).

Which newspaper's result is likely to be the best for predicting the level of support for the President if the election is held on January 25? Give two reasons to support your answer.



THE BEST CAR

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:

3 points = Excellent
2 points = Good
1 point = Fair

Question 1: THE BEST CAR

M704Q01

To calculate the total score for a car, the car magazine uses the following rule, which is a weighted sum of the individual score points:

$$\text{Total Score} = (3 \times S) + F + E + T$$

Calculate the total score for Car "Ca". Write your answer in the space below.

Total score for "Ca": .....

Question 2: THE BEST CAR

M704Q02

The manufacturer of car "Ca" thought the rule for the total score was unfair.

Write down a rule for calculating the total score so that Car "Ca" will be the winner.

Your rule should include all four of the variables, and you should write down your rule by filling in positive numbers in the four spaces in the equation below.

Total score = ...× S + ...× F + ...× E + ...× T.

M704_THE BEST CAR 2

THE BEST CAR

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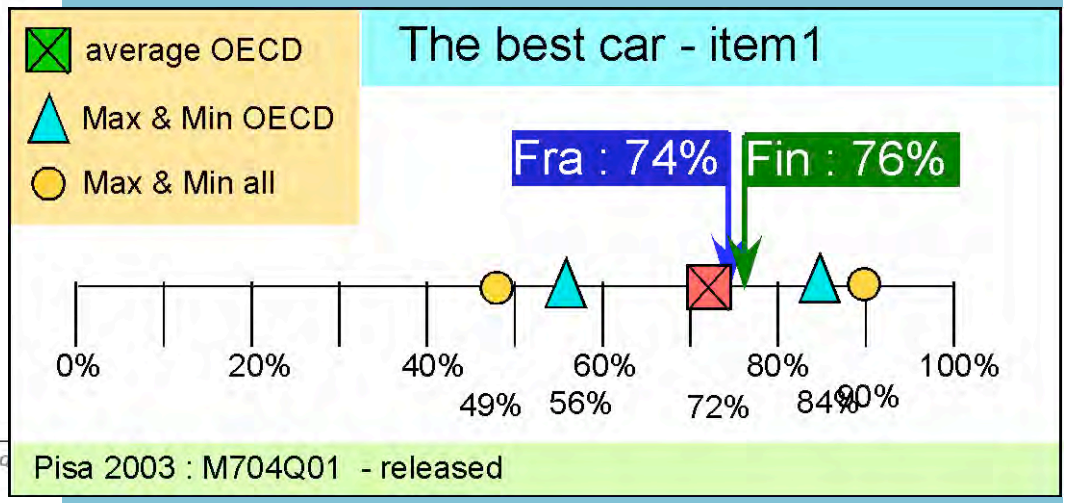
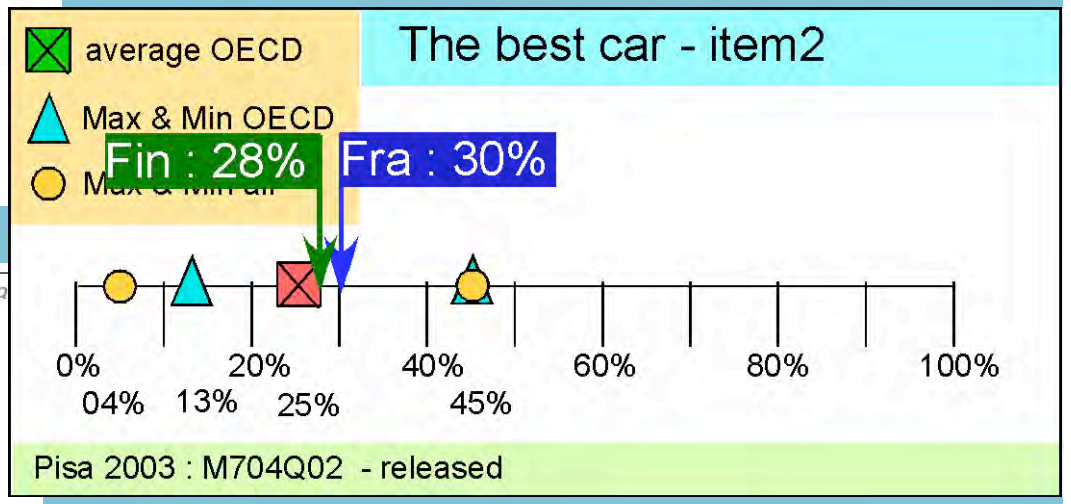


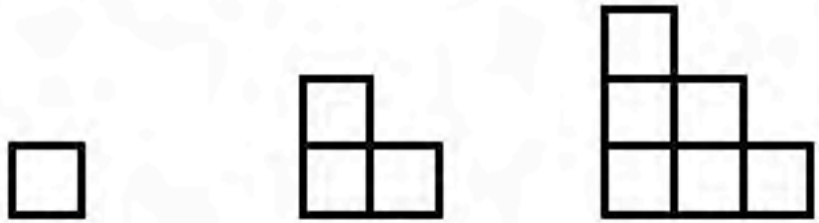


STEP PATTERN

Question 1: STEP PATTERN

M806Q01

Robert builds a step pattern using squares. Here are the stages he follows.



Stage 1 Stage 2 Stage 3

As you can see, he uses one square for Stage 1, three squares for Stage 2 and six for Stage 3.

How many squares should he use for the fourth stage?

Answer: ..... squares.

STEP PATTERN

Question 1: STEP PATTERN M806Q01

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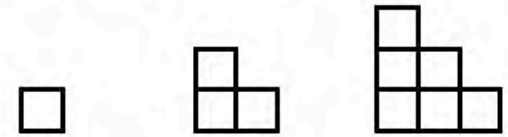

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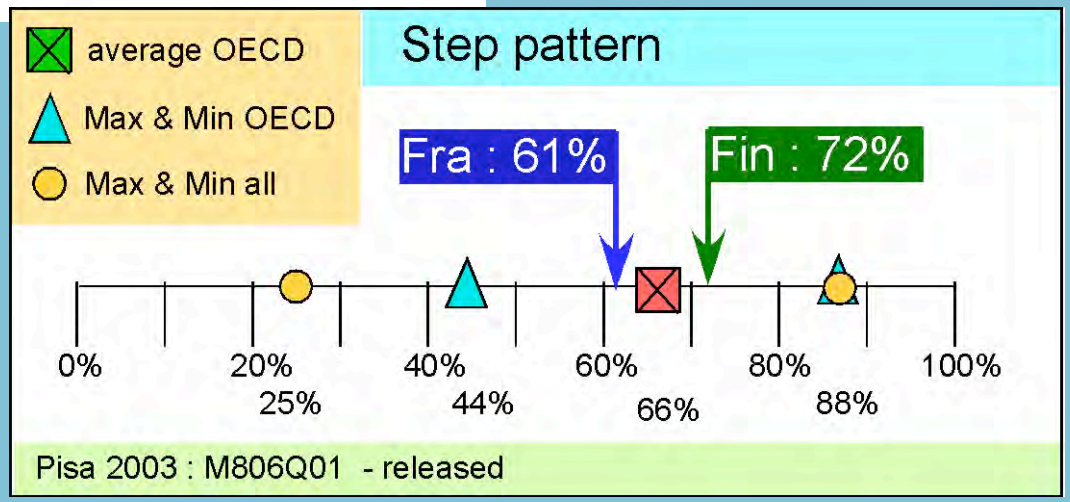
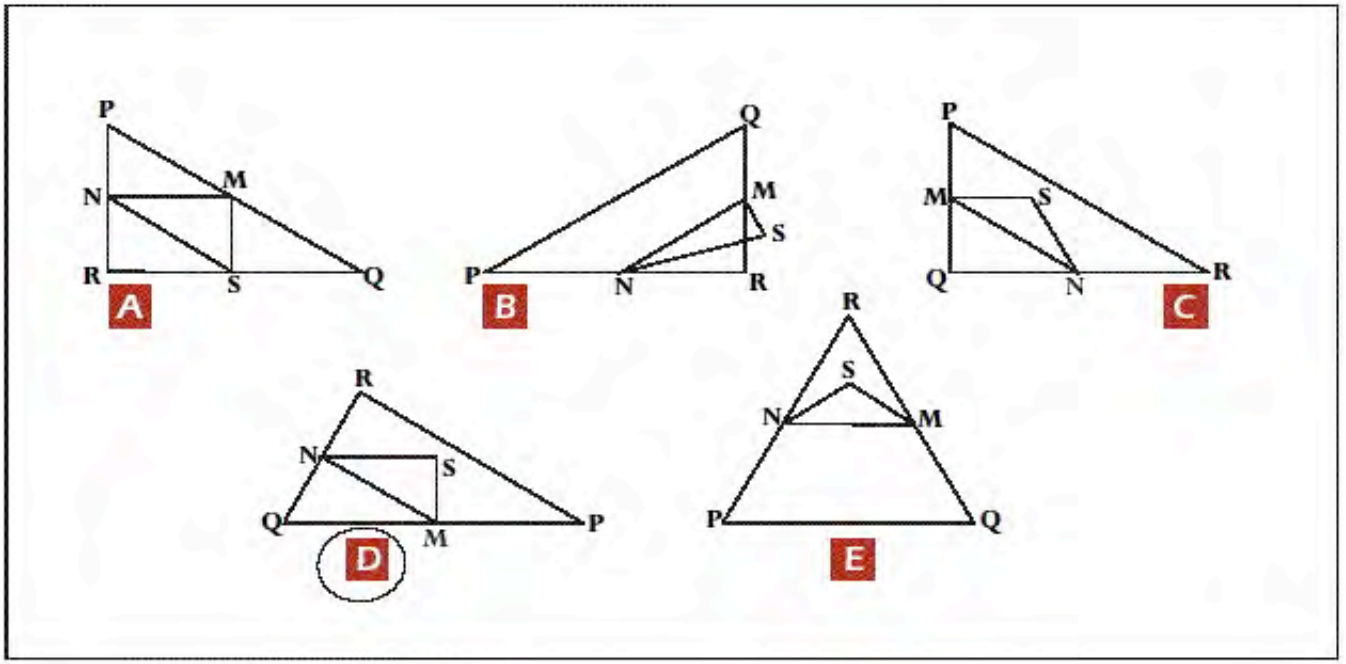


Triangles

Circle the one figure below that fits the following description.

Triangle PQR is a right triangle with right angle at R. The line RQ is less than the line PR. M is the midpoint of the line PQ and N is the midpoint of the line QR. S is a point inside the triangle. The line MN is greater than the line MS.



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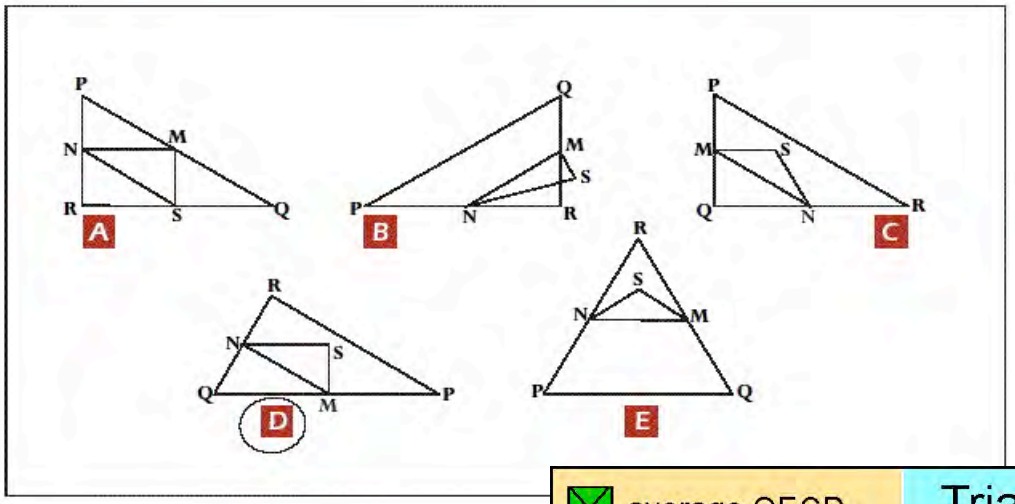
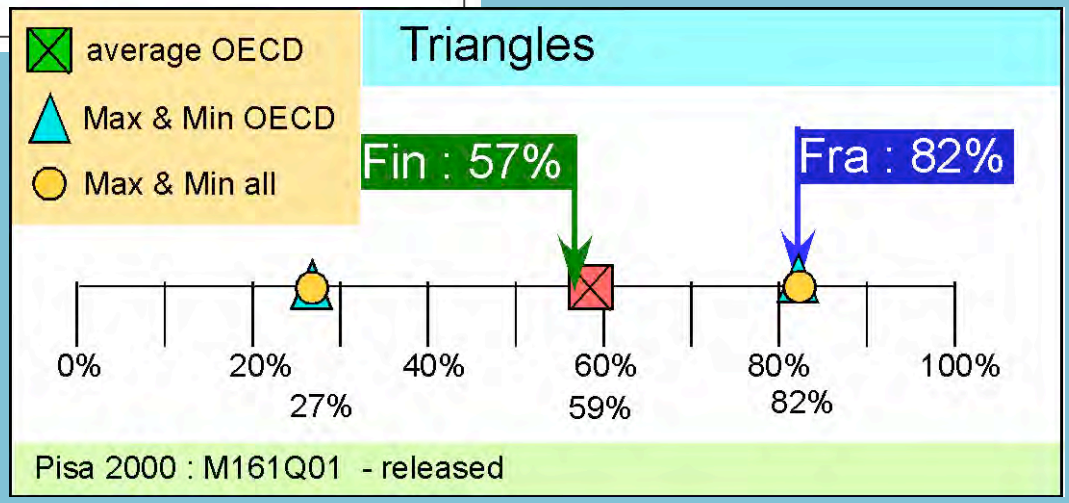
Triangle PQR is a right triangle with right angle at R. The line RQ is less than the line PR. M is the midpoint of the line PQ and N is the midpoint of the line QR. S is a point inside the triangle. The line MN is greater than the line MS.

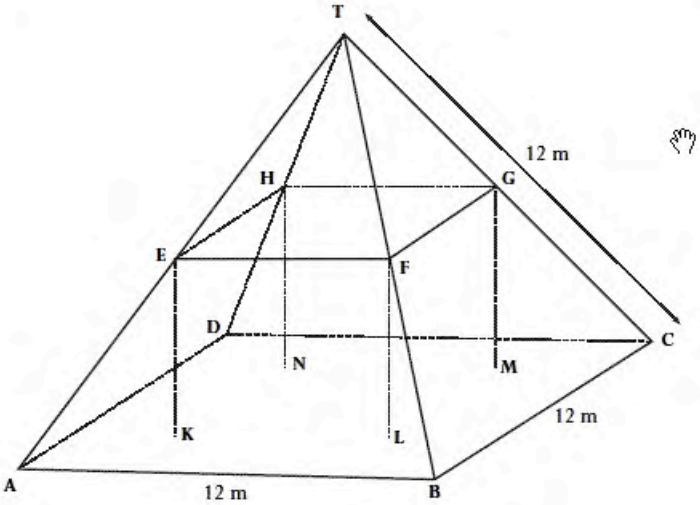




Farms

Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.

Below is a student's mathematical model of the farmhouse **roof** with measurements added.



Question 10: FARMS (M037Q01)

***Process**: Competency class 1 (Reproduction, definitions and computations)*
***Content**: Space and shape*
***Situation**: Occupational*

Calculate the area of the attic floor ABCD.

The area of the attic floor ABCD = _____ m^2

Question 11: FARMS (M037Q02)

***Process**: Competency class 2 (Connections and integration for problem solving)*
***Content**: Space and shape*
***Situation**: Occupational*

Calculate the length of EF, one of the horizontal edges of the block.

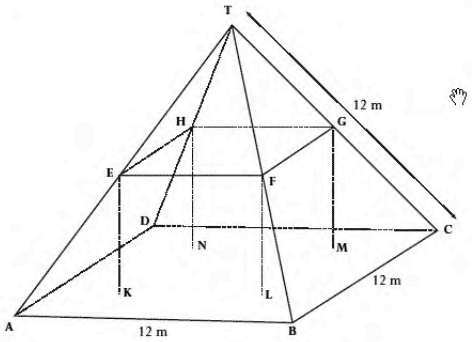
The length of EF = _____ m

M037_FARMS_PISA2000 2

Farms

Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.

Below is a student's mathematical model of the farmhouse **roof** with measurements added.



***Question 10: FARMS** (M037Q01)*

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***Situation**: Occupational*

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The area of the attic floor ABCD = _____ m²

***Question 11: FARMS** (M037Q02)*

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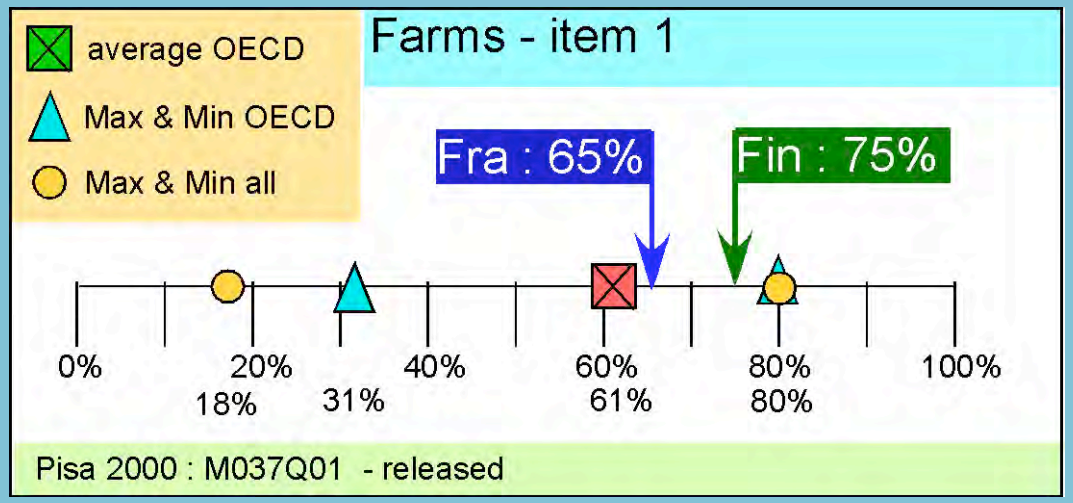
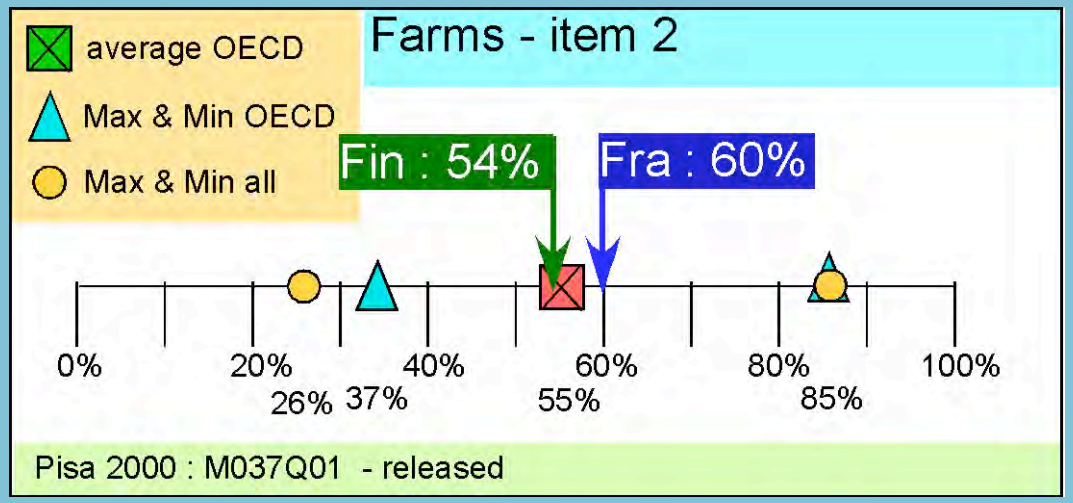
Calculate the length of EF, one of the horizontal edges of the block.

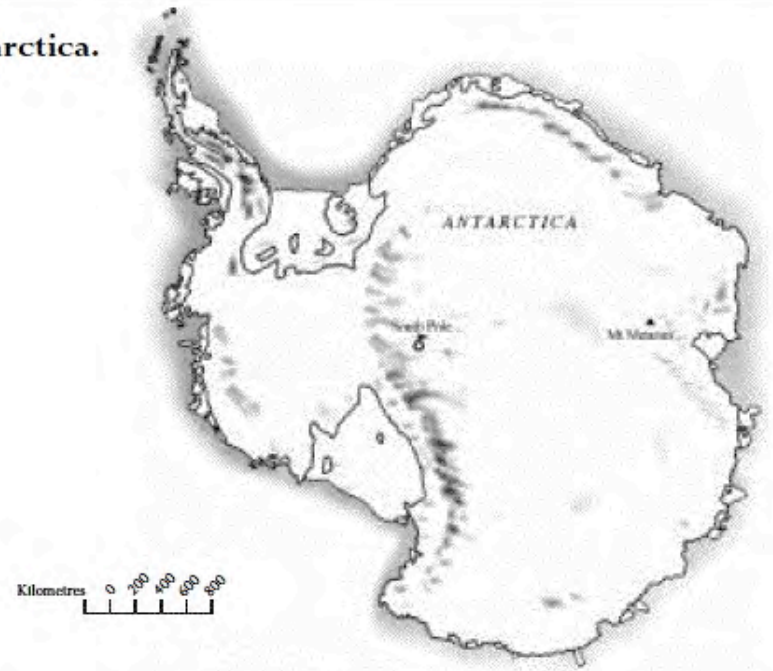
The length of EF = _____ m





Continent area

Here is a map of Antarctica.



***Question 4: CONTINENT AREA** (M148Q02)*

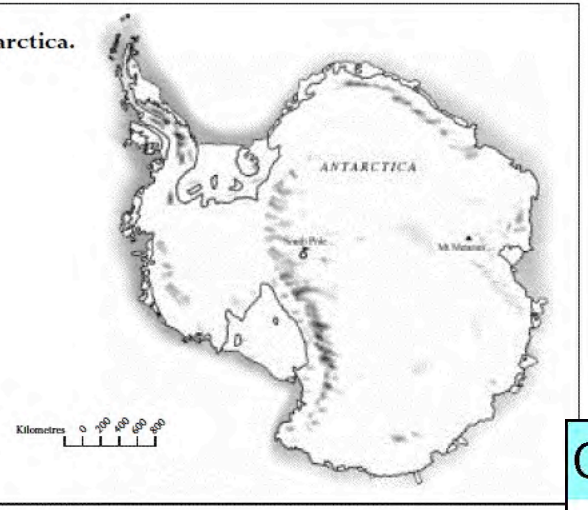
***Process**: Competency class 2 (Connections and integration for problem solving)*
***Content**: Space and shape*
***Situation**: Personal*

Estimate the area of Antarctica using the map scale.
Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation)

M148_CONTINENT AREA_PISA2000_2 + EVAPM

Continent area

Here is a map of Antarctica.



***Question 4: CONTINENT AREA** (M148Q02)*

***Process:** Competency class 2 (Connections and integration for problem solving)*
***Content:** Space and shape*
***Situation:** Personal*

Estimate the area of Antarctica using the map scale.
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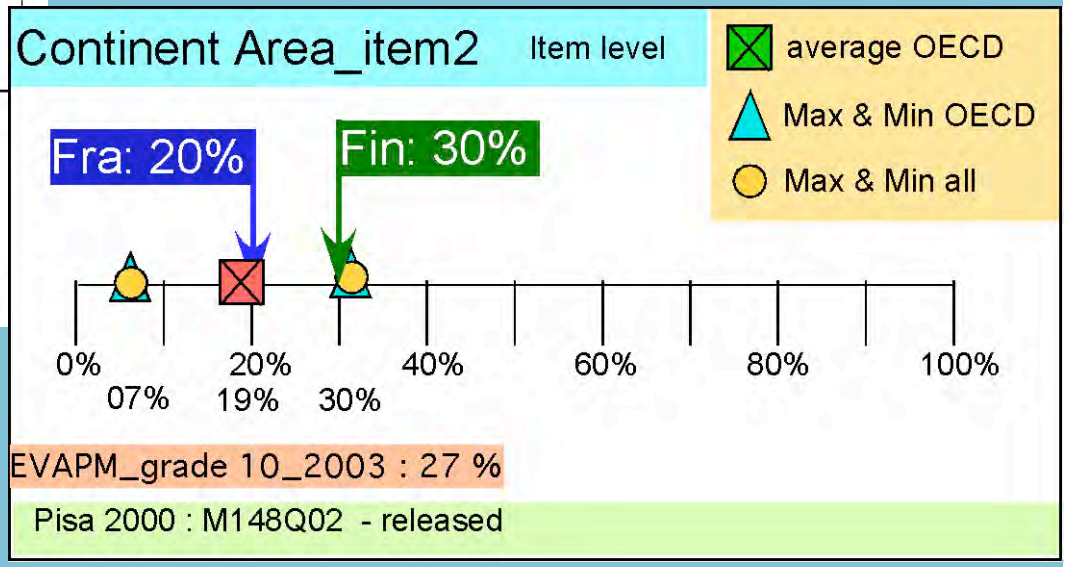
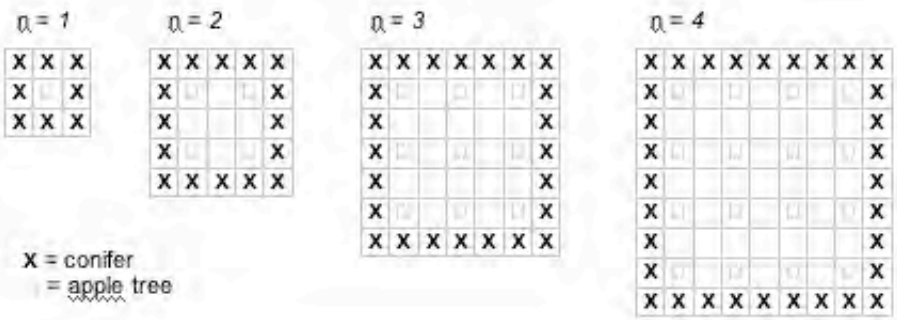


APPLES

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



Question 1: APPLES

M136Q01- 0

Complete the table:

n	*Number of apple trees*	*Number of conifers*
1	1	8
2	4	
3		
4		
5		

Question 2: APPLES

M136Q02- 00 11 12 13 14 15 99

There are two formulae you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2

Number of conifers = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifers. Find the value of n and show your method of calculating this.

.....

.....

Question 3: APPLES

M136Q03- 01 02 11 21 99

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers? Explain how you found your answer.

.....

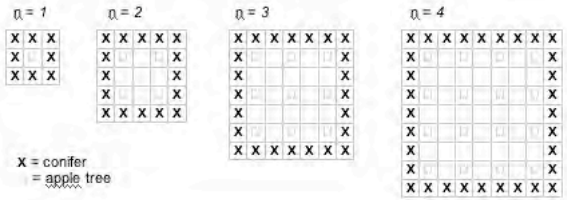
.....

M136_APPLES_PISA2000_2 + EVAPM

APPLES

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



X = conifer
= apple tree

Question 1: APPLES

Complete the table:

n	Number of apple trees	Number of conifers
1	1	8
2	4	
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4		
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Question 2: APPLES

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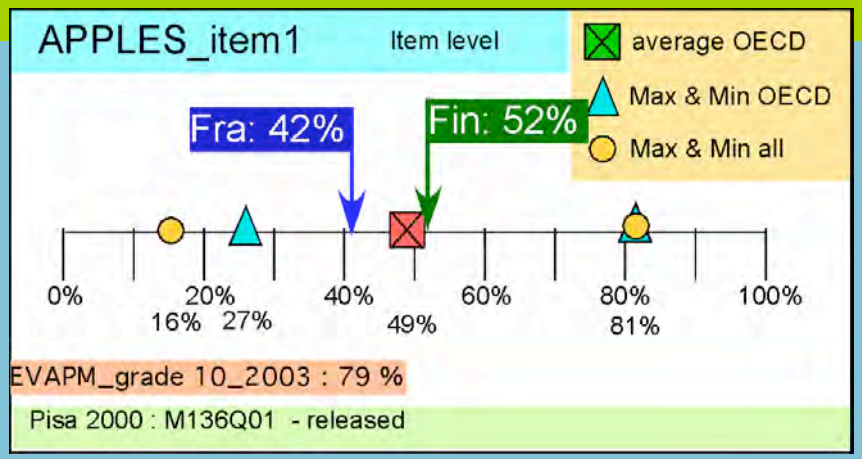
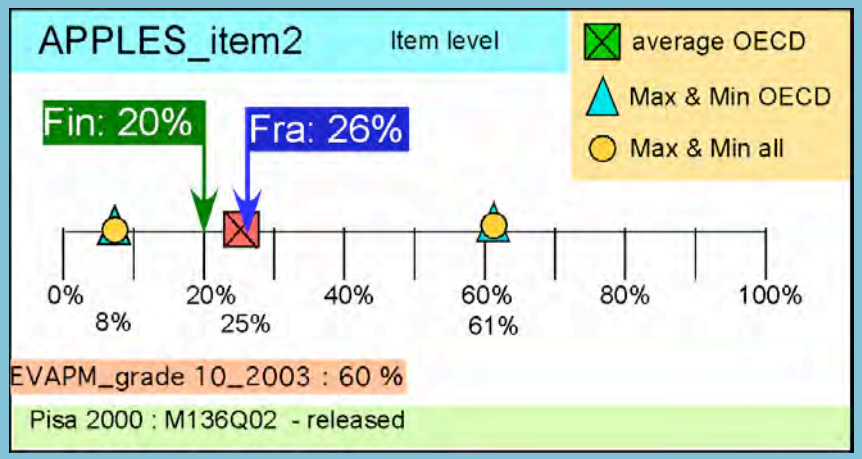
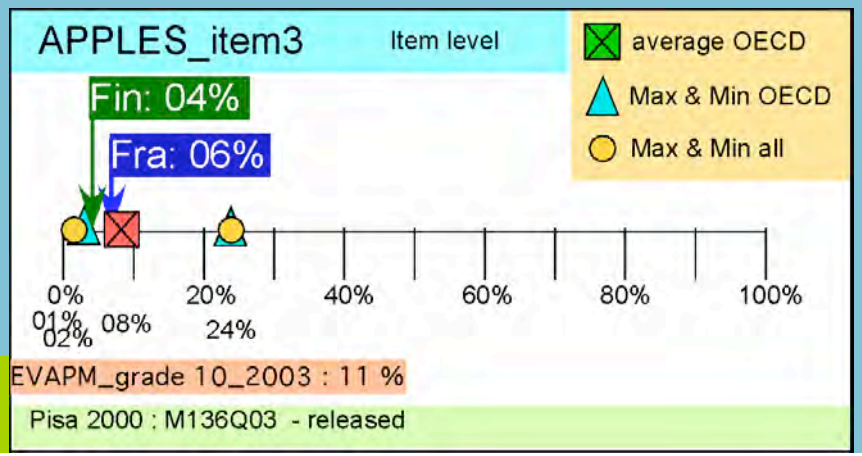
.....





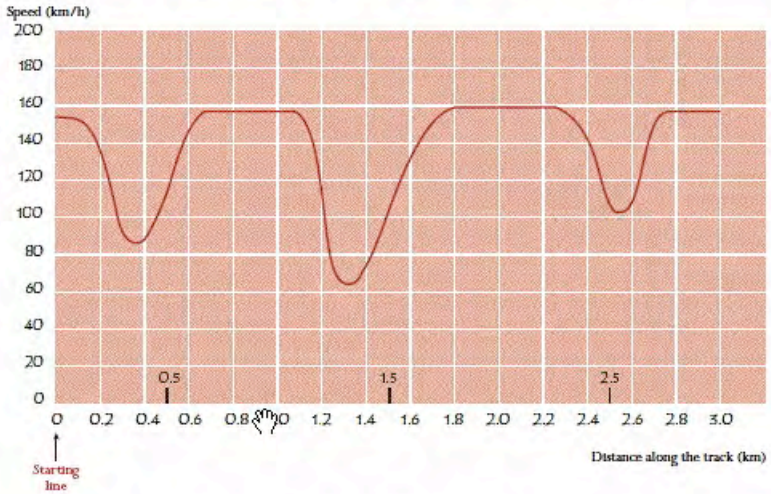


M159_SPEED OF A RACING CAR_PISA2000_1

Speed of a racing car

This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.

Speed of a racing car along a 3 km track (second lap)



Source: In memory of Claude Janvier, who died in June 1998. Modified task after his ideas in Janvier, C. (1978): The interpretation of complex graphs – studies and teaching experiments. Accompanying brochure to the Dissertation. Univrsity of Nottingham, Shell Centre for Mathematical Education, Item C-2.

The pictures of the tracks are taken from Fischer, R & Malle, G. (1985): *Mensch und Mathematik*. Bibliographisches Institut: Mannheim-Wien-Zurich, 234-238.

Question 5: SPEED OF RACING CAR (M159Q01)

Process: *Competency class 2 (Connections and integration for problem solving)*
Content: *Change and relationships*
Situation: *Scientific*

What is the approximate distance from the starting line to the beginning of the longest straight secti of the track?

A 0.5 km.
(B) 1.5 km.
C 2.3 km.
D 2.6 km.

Question 6: SPEED OF RACING CAR (M159Q02)

Process: *Competency class 1 (Reproduction, definitions and computations)*
Content: *Change and relationships*
Situation: *Scientific*

Where was the lowest speed recorded during the second lap?

A At the starting line.
B At about 0.8 km.
(C) At about 1.3 km.
D Halfway around the track.

Question 7: SPEED OF RACING CAR (M159Q03)

Process: *Competency class 1 (Reproduction, definitions and computations)*
Content: *Change and relationships*
Situation: *Scientific*

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

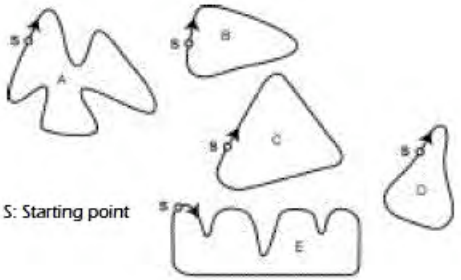
A The speed of the car remains constant.
(B) The speed of the car is increasing.
C The speed of the car is decreasing.
D The speed of the car cannot be determined from the graph.

Question 8: SPEED OF RACING CAR (M159Q05)

Process: *Competency class 2 (Connections and integration for problem solving)*
Content: *Change and relationships*
Situation: *Scientific*

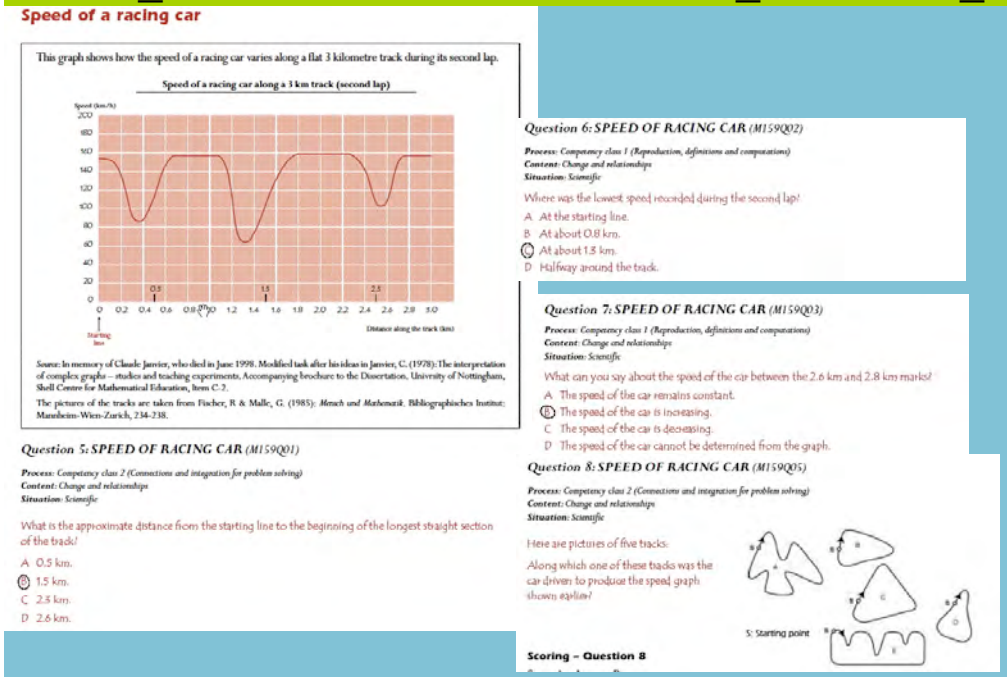
Here are pictures of five tracks:

Along which one of these tracks was the car driven to produce the speed graph shown earlier?



Scoring – Question 8

M159_SPEED OF A RACING CAR_PISA2000_2



Speed of a racing car

This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.

Speed of a racing car along a 3 km track (second lap)

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***Question 5: SPEED OF RACING CAR** (M159Q01)*

Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships
Situation: Scientific

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A 0.5 km.
B 1.5 km.
C 2.5 km.
D 2.6 km.

***Question 6: SPEED OF RACING CAR** (M159Q02)*

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Change and relationships
Situation: Scientific

Where was the lowest speed recorded during the second lap?

A At the starting line.
B At about 0.8 km.
C At about 1.5 km.
D Halfway around the track.

***Question 7: SPEED OF RACING CAR** (M159Q03)*

Process: Competency class 1 (Reproduction, definitions and computations)
Content: Change and relationships
Situation: Scientific

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

A The speed of the car remains constant.
B The speed of the car is increasing.
C The speed of the car is decreasing.
D The speed of the car cannot be determined from the graph.

***Question 8: SPEED OF RACING CAR** (M159Q05)*

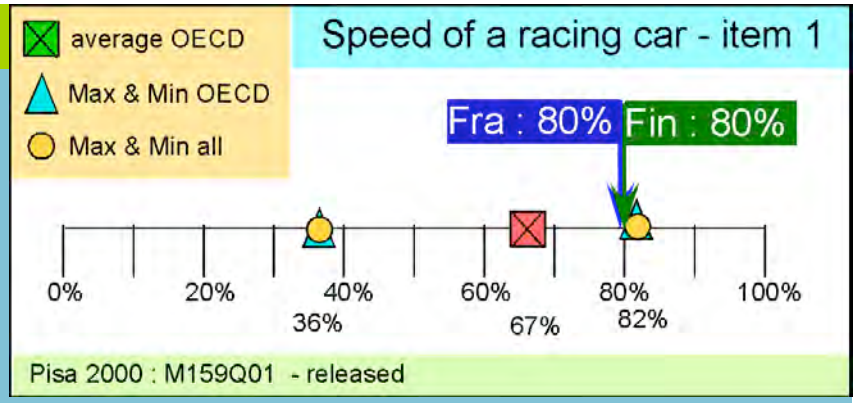
Process: Competency class 2 (Connections and integration for problem solving)
Content: Change and relationships
Situation: Scientific

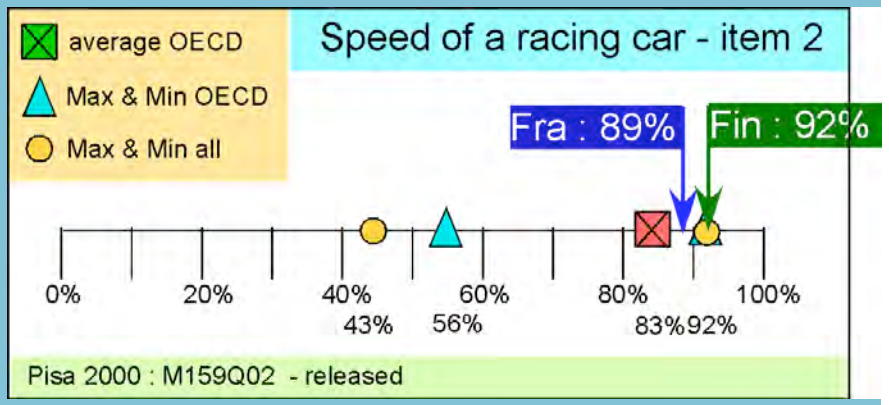
Here are pictures of five tracks.

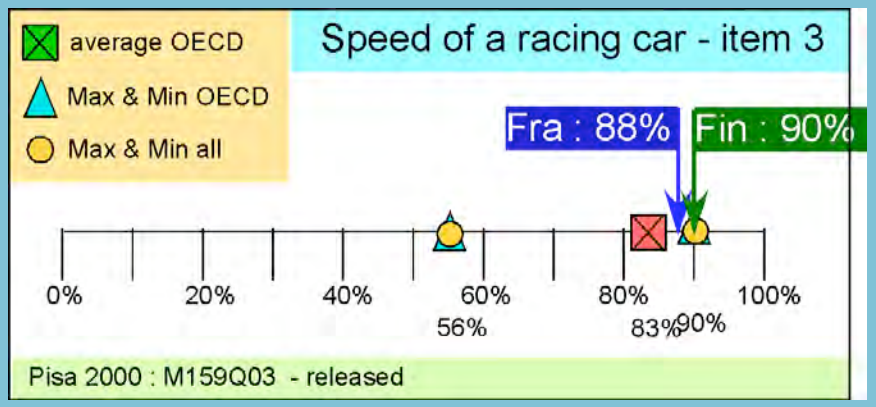
Along which one of these tracks was the car driven to produce the speed graph shown earlier?

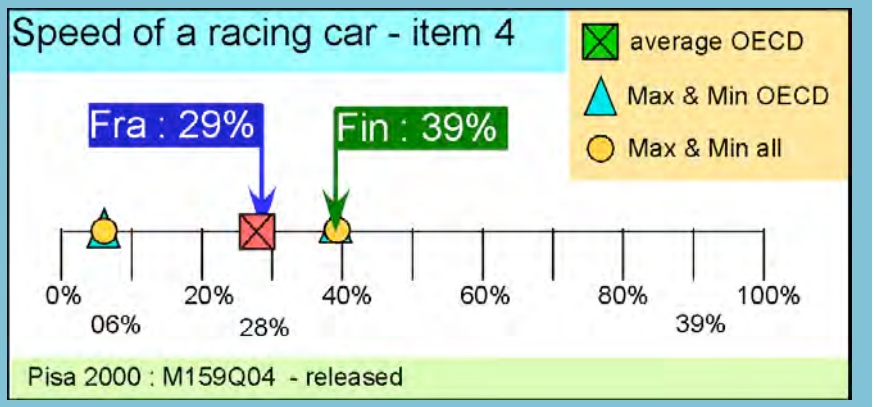
S: Starting point

Scoring – Question 8









Addresses and contacts

APMEP : 26 rue Duméril 75013 PARIS

http://www.apmep.asso.fr/

With full access to EVAPM databases

European Mathematical Society

Reference Levels in School Mathematics Education in Europe

http://www.emis.de/projects/Ref/

http://www-irem.univ-fcomte.fr/INTERNAT.HTM

IREM – Université de Franche-Comté –

La Bouloie – 25030 BESANCON CEDEX – FRANCE

http://pegase.univ-fcomte.fr/

Antoine Bodin : bodin.antoine@nerim.fr , antoinebodin@mac.com