What does PISA really assess? What it doesn't? A French view[1]

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A French language version is available as well as two presentations used for the Conference (also in English and in French - see addresses in page « references »)

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1. Summary

This paper puts aside many important aspects of the PISA design to focus on the external validity issue of its mathematics questions.

First it seeks to position the PISA item contents against the French mathematical syllabus, trying to identify the overlap of them both.

Then it tries to compare the PISA mathematical cognitive demands and competency levels with those implied in some French assessment and examination settings.

Underlining some differences between the general PISA design and the French mathematical curriculum and school culture, it also tackles the PISA mathematical items epistemological and didactical validity issues.

Cet article laisse de côté de nombreux points importants des études PISA pour se centrer sur l'examen de la validité externe des questions du domaine mathématique.

Tout d'abord il cherche à situer les contenus mathématiques des questions par rapport au curriculum français, et essaie de quantifier le recouvrement par PISA de ce recouvrement.

Ensuite il tente de comparer la complexité cognitive des questions mathématiques de PISA avec celle des questions d'examens et d'évaluations courantes en France.

Pointant des différences entre les conceptions liées aux études PISA et les attendus du curriculum mathématique et de la culture scolaire de notre pays, il soulève des questions relatives à la validité épistémologique et didactique de l'étude.

2. Introduction

The PISA studies have been organised by the OECD, which as everyone knows is an organisation devoted to the world economical development. The main reason that led this organisation to undertake such a study lies in a strong belief that good education is the key to better development.

We will not examine in this paper the value of this belief, nor the economical and political implications of the studies.

At the same time we will accept the idea that the PISA mathematics framework is consistent with the general PISA design and that the mathematics test development has been made as faithfully and as accurately as possible (and personally I think so). That's for the internal validity issue.

Plenty of documents have been written and displayed all around the world about the PISA studies. Part of them directly issued by the OECD and by the PISA consortium and many others by officials, research teams... media, in participating countries.

So, the information is rich and contrasted. Most of the documents are public and the OECD has done its utmost to allow scholars and people interested to have full access to the PISA general design, the frameworks, the complete database, as well as to the international reports.

Far from the horse race, often denounced, on which too much interest is generally given, the PISA studies produce quality data of interest for a huge range of complementary studies, from politics to didactics.

Many international and nationa[3]l analyses have been undertaken which try to draw from processed data, as well as from raw data, the information of interest for all kind of people concerned by educational matters.

Meanwhile not much effort has been made until now to examine the set of mathematics questions from an external point of view and try to more efficiently understand what they really assess and to which degree they may be viewed as epistemologically and didactically consistent. More research on those points would condition possible implication for teaching and for teachers.

This paper will only seek to examine the PISA external validity, limited to its mathematical part, and that, from a French point of view (French, as related to the French mathematics curriculum, French customary assessment settings, etc.).

3. Intended and implemented PISA assessment focus

First it seems important to recall that PISA doesn't pretend to assess the general quality of the educational systems examined. Regarding our topic, it doesn't pretend to assess the general mathematical proficiency, but simply concentrates in what the OECD juge essential for the normal life of any citizen (the so called mathematical literacy).

Let's quote the official report :

"PISA seeks to measure how well young adults, at age 15 and therefore approaching the end of compulsory schooling, are prepared to meet the challenges of today's knowledge societies. The assessment is forward-looking, focusing on young people's ability to use their knowledge and skills to meet real-life challenges, rather than merely on the extent to which they have mastered a specific school curriculum. This orientation reflects a change in the goals and objectives of curricula themselves, which are increasingly concerned with what students can do with what they learn at school, and not merely whether they can reproduce what they have learned."[4]

Anyway, individual students who don't correctly answer the PISA mathematics questions seem deemed to personal trouble and countries that don't perform well are viewed as badly preparing their youths for their future life.

So, PISA doesn't assess that whole mathematical knowledge acquired in schools but at least a part of it.

Herein I will try to identify more clearly the part really assessed by PISA and to relate this part to the whole French school mathematics for 15 year olds. The relations between this "literacy" part and the whole being a tricky question that will lead to raising epistemological and didactical complex issues.

But first let's have a look at the way the PISA material is linked to the French mathematics curriculum.

[4] OCDE (2004) : Learning for Tomorrow's World. First Results from PISA 2003. P.20

4. A comparison of the PISA mathematics item content with the current French mathematical syllabus.

For the moment, let us limit ourselves to the French syllabus which most of the 15 years old French students have studied. I mean the French college syllabus from grade 6 to grade 9 (French “sixième“ to “troisième”). At 15 some French students attend High school while others are still lagging behind, a few others are in special education, but on the whole, more that 85 % of the age group have studied this syllabus[5].

The reader will find in annexe 7 a presentation of this syllabus with indication of the topics that are addressed by at least one PISA mathematics question.

Annexe 3 shows a list of analysed PISA questions.

Here we should recall that part of the PISA questions have been secured for future use. In this paper I will only quote some of the released questions while most of the used questions have nevertheless been taken into account in the analysis.

Finally we find that the PISA questions cover about 15% of the French syllabus met by more that 85% of the 15 years old French students. That shows beyond any doubt the marginal focus of the PISA questions (but marginal doesn’t mean not important!).

At the same time those 15% represent only about 75% of the PISA mathematics questions. That means that about 25% of these questions don’t fit the French curriculum. It is particularly the case for many questions of the uncertainty field, but it is also the case for questions not directly linked to our current curriculum (such as some combinatorial questions).

But, certainly, an assessment setting can never cover 100% of any curriculum. To go further it seems useful to compare the PISA material with some customary French examinations.

5. A comparison of the PISA mathematics item content with some French examination and assessment settings at 15 years old.

5.1. Comparison with the grade 9 national examination.

We choose to analyse in the same way an issue of the mathematics form of the national examination taken by all students at the end of French middle school (grade 9).

Annexe 7 shows the corresponding curriculum coverage, while the examination form is displayed in annexe 5 and an analysis chart appears in annexe 4.

Here we find that the “Brevet” examination form covers about 35% of the syllabus.

What is more, the coverage by PISA focuses more on the syllabus for grade 6 and 7, while the coverage by the “brevet” concerns mainly the grade 8 and 9 syllabus.

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[5] As a fact the 15 year old official target is somewhat misleading. Let’s quote the PISA technical report (page 46) :

« The 15-year-old international target population was slightly adapted to better fit the age structure of most of the northern hemisphere countries. As the majority of the testing was planned to occur in April, the international target population was consequently defined as all students aged from 15 years and 3 (completed) months to 16 years and 2 (completed) months at the beginning of the assessment period. »

That leads to 59,1% of the French students who took the tests were in High schools, at grade 10 or for a few of them, grade 11.

But the Brevet examination is a poor illustration of the French intended curriculum (programmes et instructions officielles) as well as the teachers' aims and teaching practices. The Brevet is well known for shrinking the objectives and preparing the brevet is not viewed as a good way to prepare further High School studies.

5.2. The EVAPM studies.

In the following sections I will refer to a series of large-scale studies organised in the "EVAPM Observatory".

EVAPM is a research project conducted for 20 years by the Mathematics Teacher Association (APMEP) and the National Institute for Pedagogical Research (INRP), to follow the evolution of the French mathematics curriculum and especially the attained curriculum, from grade 6 to grade 12.

Being strongly linked to the teachers, and implying them in the test development process, the EVAPM studies obviously reflect their beliefs and intentions.

In recent EVAPM studies there was strong teacher resistance when we tried to introduce some PISA questions. Most of the questions were considered as not fitting the curriculum and many of them were considered as culturally biased.

It's not relevant to talk here of curriculum coverage as the EVAPM studies try to be comprehensive (100% coverage).

But we will latter use them to compare the cognitive demands of PISA and other French assessment setting.

5.3. Comparison of cognitive demands.

To compare the cognitive demands of mathematics assessment questions, we use a cognitive taxonomy, of which the main categories are:

- A Knowing and recognising…
- B Understanding...
- C Applying…
- D Creating…
- E Evaluating…

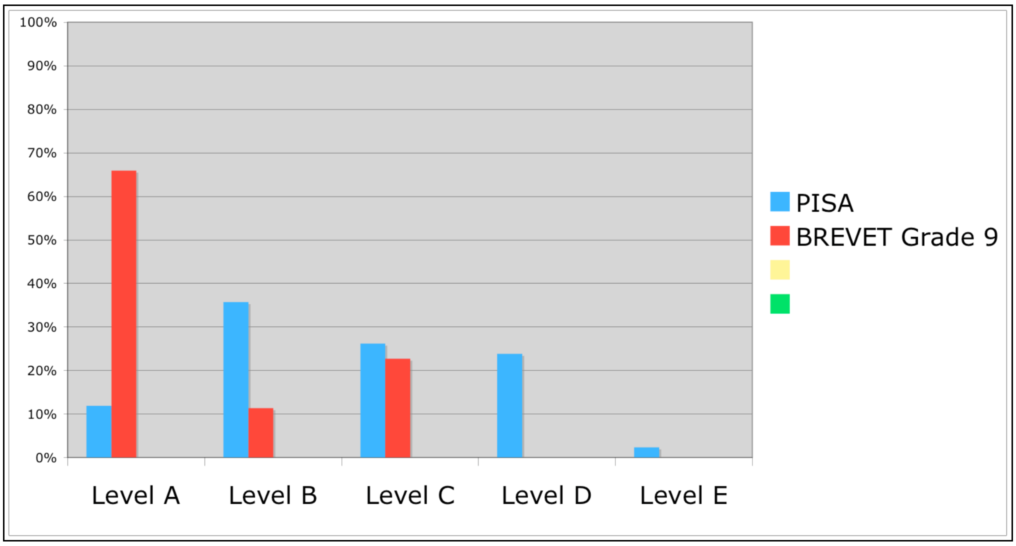
See annexe 4 for expansion of this taxonomy.

The following chart displays the PISA levels of cognitive demands along those of the Brevet examination paper already examined.

The difference is most striking: the brevet addresses mostly the recognition level, and even the classification of some items at level C (application) might be questioned (most of them are routine procedures that might better have been classified at level A).

Without doubt, the taxonomical range of the PISA items is much more balanced than the one of the French examination[6].

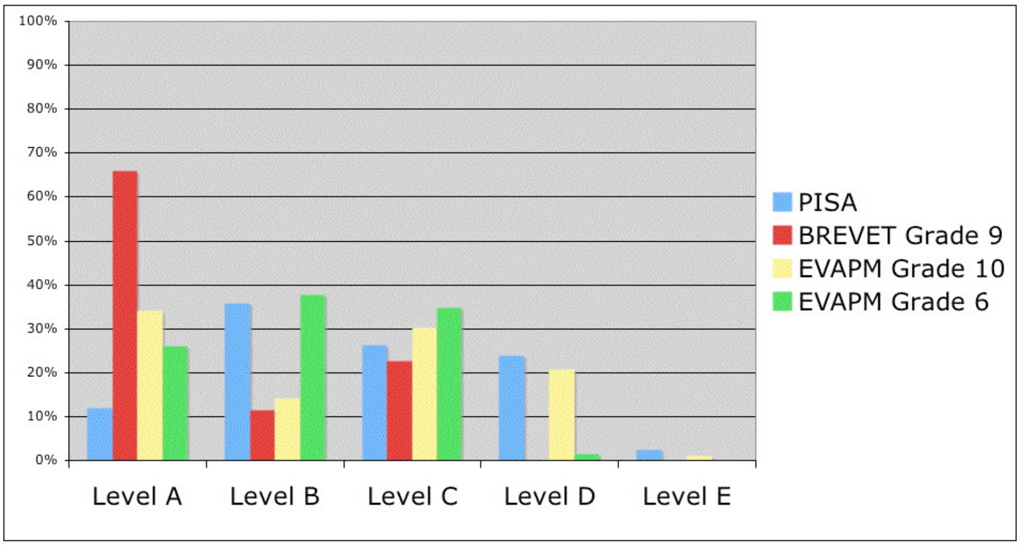
[6] Renovation of the Brevet is on the agenda. May be PISA will help ?



But, as we have already noted, the brevet doesn't correctly reflect the French real curriculum.

The following chart adds classifications obtained for two EVAPM studies (grade 10 – 2003 and grade 6 – 2005).

Here, the balance across levels is closer to PISA, at least at the same age (grade 10).



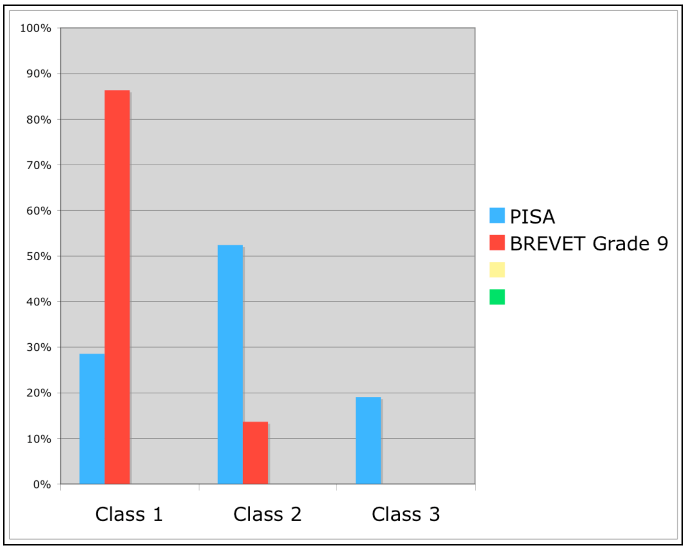
5.4. Comparison of implied range of competencies

PISA makes use of a three competency level classification:

- **Class 1: Reproduction**: "… consists of simple computations or definitions of the type most familiar in conventional mathematics assessments".
- **Class 2: Connection**: "… requires connections to be made to solve straightforward problems".
- **Class 3: Reflection**: "… consists of mathematical thinking, generalization and insight, and requires students to engage in analysis, to identify the mathematical elements in a situation and to pose their own problems".

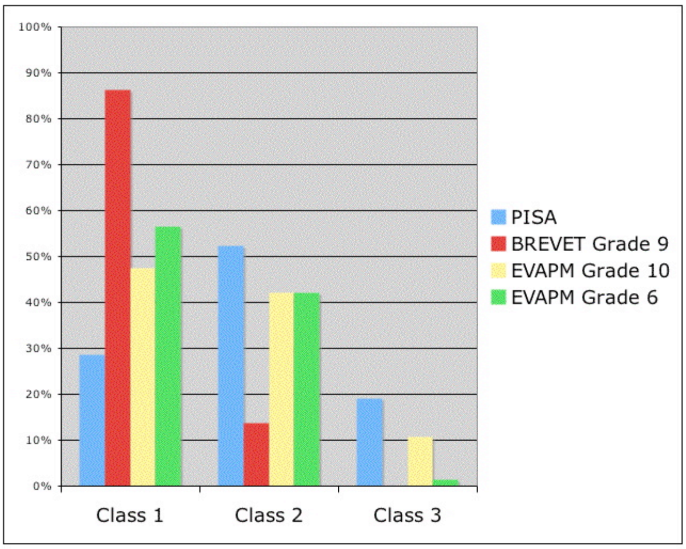
See annexe 5 for more details[7].

The following chart displays the competencies levels of the PISA items along those of the brevet.



PISA puts more than 70% of the stress on levels 2 and 3 when the Brevet examined puts less than 15% at those levels.

Here again, we can examine some EVAPM assessment settings.



The chart shows again a balance much closer to PISA for the EVAPM studies that for the national examination.

[7] Note that for EVAPM we use a competency classification originated in the Aline Robert works (see references), which, while based on other assumptions that the PISA classification, provides about the same repartition.

6. Towards some epistemological analyse

6.1. About Finland and France differences

For this paper, we had a special interest in the differences between the Finnish and French results. The overall results on a global scale (511 for France, 548 for Finland) hide the fact that this difference means a difference of .33 standard deviation on the standard normal distribution and that at a point were the density of probability is at its maximum. Not many people know that and still less understand it.

Looking to the subscales of the study (quantity, change and relationships, space and shape, uncertainty) doesn't shed any supplementary light. To help understand the observed differences it is indispensable to turn to the questions themselves and to the percentages of success in each country (or for other approaches for each of the sub groups investigated[8]).

First let's say that this examination confirms the better Finnish results – it is only the magnitude of the differences and its meaning that can be questioned.

About the magnitude, let's say that, according to the items on scope, the differences in success rates range from + 30% at the Finnish advantage to + 25% at the French advantage. The average of the differences being 3.5 % to the Finnish advantage.[9]

We observe that the differences are more important in favour of the Finnish students for the more "realistic" items and that the differences tend to turn in favour of the French students for more abstract or formal questions (compare for instance, below, the results of "Apples item 1" with the results of "Apples item 3". But the case seems general).

It seems important to note that the difference in results between Finland and France would totally disappears if the 10 % French students less successful (the 10 first percentiles) were put aside.

As a fact, while only 7 % of the age group score at levels 1 or below level 1 (on a proficiency scale ranging from1 to 6), 17 % of the French students fall in those categories. That confirms the fact that France doesn't succeed well in its mathematical education for all.

The other end of the scale (level 6), concerns 7% of the Finnish students but only 3% of the French ones. This fact may be less worrying than the one concerning the low levels. Let's remember here that PISA addresses only the literacy and don't pretend to assess the general mathematical competency.

The presentation given at this conference displays all the released mathematics items for PISA 2000 and PISA 2003 with the recorded results for the OECD, for Finland and France, as well as the higher and lower percentages of success in the OECD and in the whole set of participating countries.[10]

6.2. Mathematics ?

The mathematical field may be extended or restricted according to different conceptions. Some mathematics PISA questions surprise many French mathematics teachers. They don't recognise the

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[8] We don't mention in this paper the gender question but our analyse points out some gender bias at least for some countries. As the overall results are weaker for girls than for boys in all countries but two, the question opens for more examination. But others sub groups might also be worth to be scrutinized.

[9] That is only a rude estimate – only 41 items have been put into account.

[10] This presentation may be download from the APMEP or the SMF websites.

mathematics they strive to teach. At the same time they recognise the usefulness of the knowledge implied by these questions. Same thing for mathematicians: the insertion of many mathematics PISA questions in the theoretical mathematical building is not obvious for them.

Numbers, quantity, shapes, space, uncertainty,… are modelised in mathematics theories, but are also used in common situation, using common sense and common language.

Conversely, PISA can't help to use normal language to display its questions. In some case, understanding a text, which is in no way a mathematical text, is the main difficulty the students have to face. Certainly, that is also part of the mathematical process, but the true mathematical work begins once the problem is fully understood. Here the "devolution" process is not controlled and it is never sure if it were the dressing up or the wording that prevented the students from solving the problem or the (often) trivial mathematical difficulty.

Correlation studies between individual results in reading literacy and mathematical literacy would be useful to understand better this point.

Numbers, quantity, etc. appears also in the PISA reading questions, in the science questions and in the problem solving questions. It is not always obvious whether a PISA question should be allocated to a branch of the study instead of another one. Especially, some problem-solving question could have been analysed and gathered with the set of mathematics questions. We were not able to do that but it's an idea kept in reserve for latter.

6.3. The Apples example

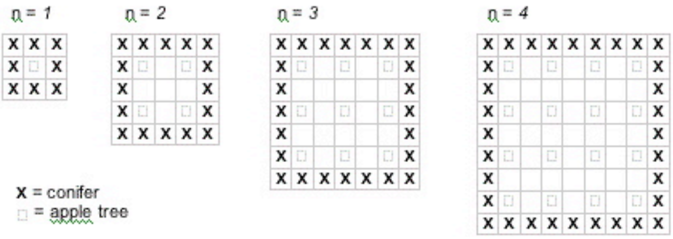
This question is typical of realistic mathematics (and authentic assessment) that the OECD seeks both to assess and to promote. In this context, a good question must open to the process thus described in the framework:

a. Starting with a problem situated in reality.
b. Organising it according to mathematical concepts.
c. Gradually trimming away the reality through processs such as making assumptions about which features of the problem are important, generalising and formalising (…), and transforming the problem into a mathematical problem that faithfully represents the situation.
d. Solving the mathematical problem.
e. Making sense of the mathematical solution in terms of real situation.

APPLES

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

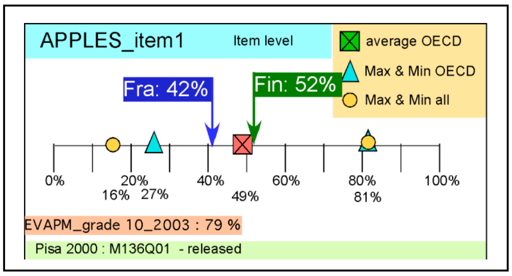
Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



Question 1: APPLES M136Q01- 01 02 11 12 21 99

Complete the table:

n	*Number of apple trees*	*Number of conifers*
1	1	8
2	4	
3		
4		
5		



Question 2: APPLES M136Q02- 00 11 12 13 14 15 99

There are two formulae you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2

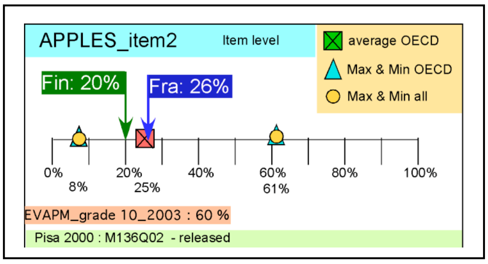
Number of conifers = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifers. Find the value of n and show your method of calculating this.

....

....

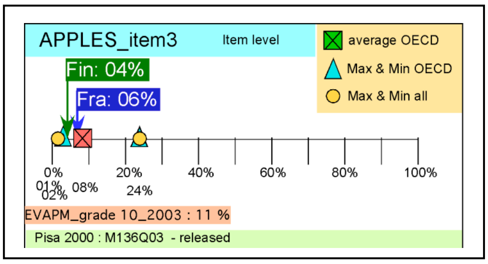


Question 3: APPLES M136Q03- 01 02 11 21 99

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers? Explain how you found your answer.

....

....



For item 1 the main thing is to understand the situation and after that being able to extrapolate a pattern. Just by counting the first four lines in the chart can be completed. For the fifth one the student can either extend the drawing and then count, or identify a number pattern in the completed chart.

The 10% difference between French and Finnish students illustrates the French students relative lack of confidence or lack of initiative. They don't have a mathematical procedure on hand to treat the question and that stops some of them.

Conversely, those who overcome this first difficulty perform much better in the second item that there Finnish counterpart (62% instead of 38% - ratio computed just from those who successfully completed item 1). That also seems to be rather general.

For this item, the mathematisation is quite obvious and leads to an equation to be solved: $n^2 = 8n$.

French students are used to solve this type of equation (often though in a formal, non realistic, context). We may even suppose that many of them have used a correct mathematical method, by which I mean, factorizing [n(n – 8) = 0] and find the two values: 0 and 8. Then and only then (point e upward) eliminate the value 0 and keep the value 8.

But some students should have got the correct answer just by this not valid simplification: $n^2 = 8n \Leftrightarrow n = 8$ or n x n = 8 x n ⇔ n ~~x n~~ = 8 ~~x n.~~

Another procedure consists of extending the chart until n = 8.

Those procedures, of which one is mathematically wrong, along with other procedures, have been considered correct (full or partial credit!). That raise the epistemological issue: which mathematics are at stake? What is valued?

Let's be clear: it is not our purpose to deny the interest of the question, nor its relevance in a mathematical test, not even the legitimacy of building scales on an utilitarian point of view (which works is good!).

What is raised here is the need for complementary qualitative studies which more deeply analyse students' procedures from a mathematical point of view.

Item 3 needs to compare two variation rates. Here, that may lead to compare the growth of the derivatives of functions *f* such as $f(n) = n^2$ and *g* such as $g(n) = 8n$, and, finally to compare the second derivatives.

Here again students are not supposed to know derivatives… they should just have a sound and personal approach to the question. Several procedures are possible, that have different mathematical values, but that are considered the same.

Note that the question is by no mean trivial and it is not too surprising that so few students across the world are able to cope with it.

There is also an interesting point coming out from international studies (it was the same for TIMSS). Real mathematical difficulties, I mean difficulties linked to the concepts, which are experienced in the same way all over the world, and not linked to the dressing up or the wording.

The apples question has been used in an EVAPM study at grade 10. The results of this setting appear also in the rectangles.

The 6% success rate (France) and the 4% success rate (Finland) concern only a correct mathematical procedure. Those rates have to be compared with the 11% obtained in Japan and also with the 11% obtained by EVAPM in France at grade 10.

For all countries but one the item 3 success rate range from 2% to 12%

The only exception, Korea, with 24%, would deserve further examination.

6.4. The bookshelves example

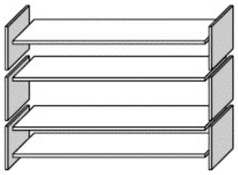
The following question is typically a case of one question that doesn't fit the current French mathematical curriculum; more precisely it would be seen as more appropriate at primary school level.

BOOKSHELVES

Question 1: BOOKSHELVES *M484Q01*

To complete one set of bookshelves a carpenter needs the following components:

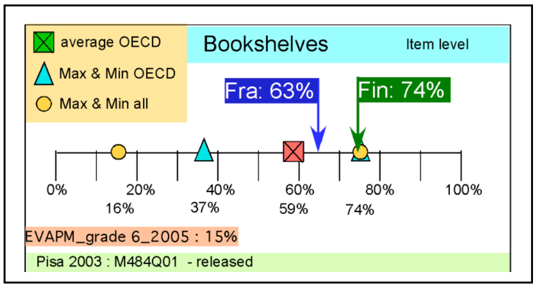
4 long wooden panels,
6 short wooden panels,
12 small clips,
2 large clips and
14 screws.



The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

Answer: ....



At the same time everyone in France (and especially French mathematics teachers) would expect students at 15 to be able to solve this question.

The success rate is more than 10% higher in Finland that in France. And that illustrates what has been said about realistic questions.

But is it a mathematical question? Or should any question using number be considered as a mathematical question? In some countries this question would be more likely asked in the technology subject matter area.

A mathematical solution would be.

$$N = Min\left(\left[\frac{26}{4}\right];\left[\frac{33}{6}\right];\left[\frac{200}{12}\right];\left[\frac{20}{2}\right];\left[\frac{510}{14}\right]\right)$$

Where N is the maximum number of bookshelves the carpenter can make, and where $[x]$ stands for the integer part of x.

Here again it is not expected of the students that they write this complex formula. In fact, they proceed by a *try and guess* method. Meanwhile if they had to prove their result, they would be forced to write down, in common language, the content as well as the meaning of this formula. That should be even more difficult that to write the symbolic formula.

Fortunately for PISA, no student thinks about using such a formula (neither would we other than in this paper!), so the international results are quite high: ranging mostly from 50% to 70%.

But is it still mathematics? Might training in this kind of realistic questions be a good preparation for more abstract mathematics? As some educational systems tend to ask teachers to put the emphasis on realistic mathematics the question is surely worth raising.

This question, along with many others, points out the weak stress given by PISA to the proof undertakings. Even explaining and justifying are not much valued by the PISA marking scheme. That makes a big difference with the casual French conception of mathematical achievement.

7. Toward some didactical analysis

The preceding remarks lead directly to the raising of didactical question.

Which sequence of teaching situations can help students to gain proficiency both in mathematical literacy (partly common sense knowledge) and in abstract and symbolic mathematics?

I know some would assume that the question is not relevant and that there is a continuum from common sense knowledge to theoretical knowledge.

On the contrary, I think, and all the work of the so called French didactics school have helped me, that ruptures are necessary and constitutive to learning. So we may fear that putting too much stress on real life and concrete situations may in return have some negative effects.

Here is an example.

7.1. The coloured candies question

This question belongs to the uncertainty field and a « probability » value is asked.

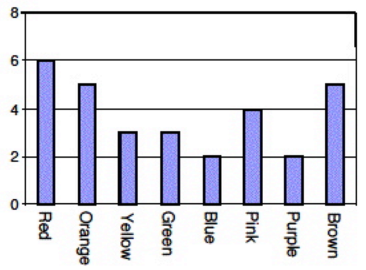
Probability is not part of the curriculum followed by 98% of the French students at age 15, meanwhile they perform at the same level as other OECD students.

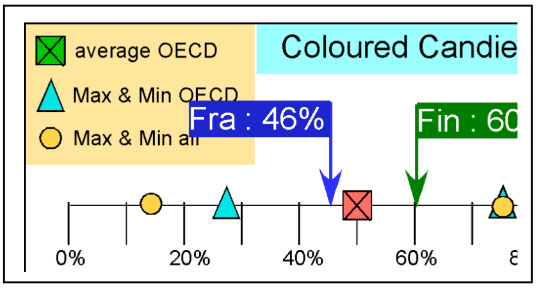
We obtained the same kind of result with TIMSS at age 13. While probability was not in the curriculum, French students performed better that others in countries were probability was considered as part of the curriculum. Other observations (EVAPM) show that when introduced to the probability concepts (at least at the beginning) students find more difficulty answering this kind of question than when they have not been taught the subject.

COLOURED CANDIES

Question 1: COLOURED CANDIES *M467Q01*

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each colour in the bag is shown in the following graph.





What is the probability that Robert will pick a red candy?

A 10%
B 20%
C 25%
D 50%

Here again we can talk of common knowledge: understanding a chart, counting the total number of candies (30), note that 6 of them are red, and finally interpret the 6 chances out of 30 as a probability value.

That is common language and preconceptions about a mathematical concept. Stressing this kind of task, especially in a MCQ format, and letting students (and many others) think they have acquired some knowledge in probability may surely lead to serious misunderstanding.

Many other questions would deserve this kind of examination.

8. Preliminary conclusion

PISA has gathered a huge amount of quality data across countries, which open ways to further research. Aside from edumetrics studies focussing on marks and scales, there is room for many interesting qualitative studies (more precisely for studies articulating quantitative and qualitative approaches).

A large amount of resources have been put in the PISA studies, as well as a great variety of commitment and competencies. If would be sad if the students were not the first beneficiaries.

In this paper, we tried to demonstrate that some precaution have to be taken, but also, in the whole, that the PISA studies are worth been taken seriously. They can bring new questions and new ideas to teachers which can help them to go ahead with a way of teaching that fits the needs of our societies as well as preserving the values of which they are depositaries.

This paper is particularly aimed to attract the scholars' attention and to justify the idea that some complementary studies should be undertaken by and within the research in the mathematics education community.

The PISA studies may help scholars in different countries to distance themselves from their national or regional place and to get a more comprehensive understanding of the teaching and the learning of mathematics, for future citizens and consumers, and above all for the advancement of mankind[11].

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ANNEXES

Annexe 1 : Taxonomy of cognitive demands for designing and analysing mathematical tasks - ordered by integrated level of complexity

Simplified version - see complete taxonomy on the Web (in French)

	Main categories		Sub-categories
A	**Knowing and recognising…**	**A1**	facts
		A2	vocabulary
		A3	tools
		A4	procedures
B	**Understanding...**	**B1**	facts
		B2	vocabulary
		B3	tools
		B4	procedures
		B5	relations
		B6	situations
C	**Applying…**	**C1**	in simple familiar contexts
		C2	in mean complex familiar contexts
		C3	in complex familiar contexts
D	**Creating…**	**D1**	as mobilizing known mathematical tools and procedures in new situations
		D2	new ideas
		D3	personal tools or procedures
E	**Evaluating…**	**E1**	as issuing judgements about external productions
		E2	as assessing one's own knowledge, process and results

Taxonomy designed by Antoine Bodin, with full acknowledgment to R. Gras seminal work as well as to W. A. Anderson later influence.

Annexe 2 : Competency classes for designing and analysing mathematical tasks - ordered by integrated level of complexity

Simplified version - see expanded version in OECD documents on the Web

Level		OECD definition	
1	**Reproduction**	The competencies in this cluster essentially involve reproduction of practised knowledge…	Reproduction
2	**Connection**	The connection cluster builds on the reproduction cluster competencies in taking problem solving to situations that are not simply routine, but still involve familiar or quasi familiar settings	Simple mathematisation
3	**Reflection**	The competencies in this cluster include an element of reflectiveness … about the processes needed or used to solve a problem. They relate to student's abilities to plan solution strategies and implement them in problem settings that contain more elements and may be more "original" (or unfamiliar) than those in the connection cluster…	Complex mathematisation (to modelisation)

Annexe 3: PISA 2003 and 2000 –Analysed Question Set

Along with some other non released questions taken into account, the whole analysis covers about 70% of the PISA material (60/85)

PISA code	Item name	Mathematical content	Taxo	C	Remarks
M037Q01	Farms 1	Pyramid – square area	B6	1	PISA2000 only
M037Q02	Farms 2	Middle of the sides of a triangle..	C1	2	PISA2000 only
M124Q01	Walking 1	Using letters and formula	C1	2	& PISA2000
M124Q02	Walking 2	Using letters and formula - Units …	B5	2	& PISA2000
M136Q01	Apple 1	Completing charts	B6	3	& PISA2000 & EVAPM
M136Q02	Apple 2	Equation	C1	2	& PISA2000 & EVAPM
M136Q03	Apple 3	Don't fit	D1	3	& PISA2000 & EVAPM
M145Q01	Cubes	Cube	B5	2	& PISA2000
M148Q02	Continent area	area	D1	3	PISA2000 only & EVAPM
M150Q01	Growing up 1	Reading graphs	B5	2	& PISA2000
M150Q02	Growing up 2	Reading graphs	B5	1	& PISA2000
M150Q03	Growing up 3	Reading graphs	B5	1	& PISA2000 - Gender bias ?
M155Q02	Number cube	Cube	B5	2	& EVAPM
M159Q01	Speed of a car 1	Interpreting graph	B6	2	PISA2000 only
M159Q02	Speed of a car 2	Reading graph	A3	1	PISA2000 only
M159Q03	Speed of a car 3	Interpreting graph	B3	1	PISA2000 only
M159Q04	Speed of a car 4	Interpreting graph	D1	2	PISA2000 only
M161Q01	Triangles	Constructing geometrical figures	B5	1	PISA2000 only
M179Q01	Robberies	Bar charts	E1	3	& TIMSS & PISA2000 & EVAPM
M266Q01	Carpenter	Perimeter of a rectangle	D1	2	& PISA2000 - Gender bias ?
M402Q01	Internet relay chat 1	Don't fit	D1	2	Gender bias ?
M402Q02	Internet relay chat 2	Don't fit	D1	3	Gender bias ?
M413Q01	Exchange rate 1	Proportionality	C1	2	
M413Q02	Exchange rate 2	Proportionality	A4	1	
M413Q03	Exchange rate 3	Proportionality	C1	2	
M438Q01	Export - 1	Bar charts	A3	1	
M438Q02	Export - 2	Circle charts - Percentage	C1	1	
M467Q01	Coloured candies	Don't fit	C1	1	Probability
M468Q01	Science test	Mean	C1	2	
M484Q01	Bookshelves	Don't fit	D1	2	& EVAPM Gender bias ?
M505Q01	Litter	Bar charts	B6	2	? Huge diff FRA-FIN
M509Q01	Earthquake	Don't fit	B5	2	Probability
M510Q01	Choice	Don't fit	D1	3	Combinatory –transtation pb
M513Q1	Test Scores	Bar graph			
M520Q01	Skateboard 1	Don't fit	C1	2	EVAPM
M520Q02	Skateboard 2	Don't fit	C1	2	
M520Q03	Skateboard 3	Don't fit	D1	3	
M547Q01	Staircase	Division	A4	1	
M555Q02	Number cubes	Cube	B5	2	
M702Q01	Support for president	Don't fit	B6	2	
M704Q01	Best car 1	Reading charts	C1	2	
M704Q02	Best car 2	Reading charts	D1	3	
M806Q01	Step pattern	Don't fit	A1	1	

PISA 2003 : 85 items released : 31
PISA 2000 : 32 items released : 11

Annexe 4 : A typical mathematical examination at middle school end.

			Taxo	Comp	Remarks
Part I - Numerical activities					
Numbers	**Ex 1**	1)	A4	1	Formal and no realistic
		2)	A4	1	id
		3)	A4	1	id
		4)	A4	1	id
Data	**Ex 2**	1)	B5	1	Pseudo-realistic
		2)	A4	1	id
		3)	C1	1	id
		4)	A2	1	id
Numbers	**Ex 3**	1) a	A2	1	Formal and no realistic
		1) b	A4	1	id
		1) c	A4	1	id
		1) d	A4	1	id
Numbers -Arthmetics	**Ex 4**	1)	C1	1	Formal and no realistic
		2)	C1	1	id
		3)	C1	1	id
Part II - Geometrical activities					
Space geometry	**Ex 1**	1) a	B1	1	No realistic
		1) b	B5	1	id
		2) a	A4	1	id
		2) b	B5	1	id
		3)	A4	1	id
		4)	A4	1	id
Plane géometry – Proof - Thalès	**Ex 2**	1)	C1	2	Formal and no realistic
		2)	C1	2	id
Plane géometry – Proof - Pythagore	**EX 3**	1)	A4	1	Formal and no realistic
		2)	A4	1	id
		3) i	A4	1	id
		3) ii	A4	1	id
Part III - Problem					
Geometry-Pythagore-Trigonometry	**Part I**	1)	A4	1	Pseudo-realistic dressing
		2) i	A4	1	id
		2) ii	A4	1	id
		3)	A4	1	id
Linear functions - inequations	**Part II**	1) a i	A3	1	id
		1) a ii	A3	1	id
		1) b	A3	1	id
		2) a	A2	1	id
		2) b	C1	2	id
		3) a	B5	1	id
		3) b	A4	1	id
		3) c	A4	1	id
Scale area - volum	**Part III**	1)	A2	1	id
		2)	A4	1	id
		3)	C1	2	id
		4) i	C1	2	id
		4) ii	C1	2	id

Annexe 5 : The examination on scope : Brevet 2005 - South of France

Wording and appearance will be counted on 4 marks out of 40..
Handheld calculators allowed.
Duration : 2 heures

1.1. Part I : Numerical activities

ACTIVITÉS NUMÉRIQUES (12 points)

Exercice 1 : (4 points)

Dans cet exercice, tous les calculs devront être détaillés.

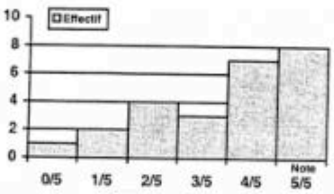
1) Calculer l'expression : $A = \frac{13}{3} - \frac{4}{3} \times \frac{5}{2}$ (donner le résultat sous sa forme la plus simple).

2) Donner l'écriture scientifique du nombre B tel que : $B = \frac{7 \times 10^{15} \times 8 \times 10^{-8}}{5 \times 10^{-4}}$.

3) Écrire sous la forme $a\sqrt{7}$ (où a est un entier) le nombre C tel que : $C = 4\sqrt{7} - 8\sqrt{28} + \sqrt{700}$.

4) Développer et simplifier : $(4\sqrt{5} + 2)^2$.

Exercice 2 : (3 points)



Voici l'histogramme des notes d'un contrôle noté sur 5 pour une classe de 25 élèves.

1) Reproduire et remplir le tableau des notes suivants.
2) Calculer la moyenne des notes de la classe ?
3) Quelle est la médiane des notes de la classe ?
4) Calculer la fréquence des notes inférieures ou égales à 3 points sur 5.

Tableau à reproduire et compléter:

Note	0	1	2	3	4	5
Effectif						
Effectif cumulé croissant						

Exercice 3 : (2 points)

Répondre aux questions suivantes. (Les calculs pourront être totalement faits à la calculatrice : on ne demande pas d'étapes intermédiaires ni de justification)

a) Donner un arrondi au centième du nombre A tel que : $A = \frac{831 - 532}{84}$.

b) Convertir 3,7 heures en heures et minutes.

c) Donner un arrondi au millième du nombre B tel que : $B = \frac{\frac{53}{51} - \frac{32}{85}}{\frac{63}{34}}$.

d) Calculer à 0,01 près $C = \sqrt{\frac{83 + 167}{158}}$

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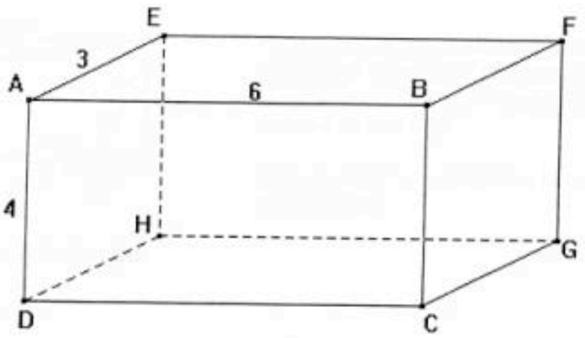
Exercice 4 : (3 points)

1) Trouver le PGDC de 6 209 et 4 435 en détaillant la méthode.
2) En utilisant le résultat de la question précédente, expliquer pourquoi la fraction $\frac{4\,435}{6\,209}$ n'est pas irréductible.
3) Donner la fraction irréductible égale à $\frac{4\,435}{6\,209}$.

1.2. part II : Geometrical activities

ACTIVITÉS GÉOMÉTRIQUES (12 points)

Exercice 1 (5 points)



ABCDEFGH est un parallélépipède rectangle. On donne AE = 3 m ; AD = 4 m ; AB = 6 m.

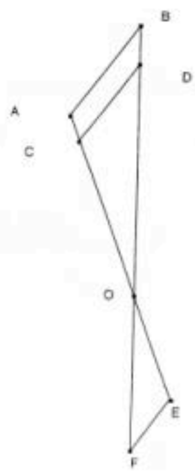
1) a) Que peut-on dire des droites (AE) et (AB) ? Le justifier.
   b) Les droites (EH) et (AB) sont-elles sécantes ?

2)
   a) Calculer EG. On donnera la valeur exacte.
   b) En considérant le triangle EGC rectangle en G, calculer la valeur exacte de la longueur de la diagonale [EC] de ce parallélépipède rectangle.

3) Montrer que le volume de ABCDEFGH est égal à 72 m^3.

4) Montrer que l'aire totale de ABCDEFGH est égale à 108 m^2.

Exercice 2 (3 points)



Sur le dessin ci-contre, les droites (AB) et (CD) sont parallèles, les points A, C, O, E sont alignés ainsi que les points B, D, O et F. (On ne demande pas de faire le dessin).
De plus, on donne les longueurs suivantes :
CO = 3 cm, AO = 3,5 cm, OB = 4,9 cm, CD = 1,8 cm, OF = 2,8 cm et OE = 2 cm.

1) Calculer (en justifiant) OD et AB.

2) Prouver que les droites (EF) et (AB) sont parallèles.

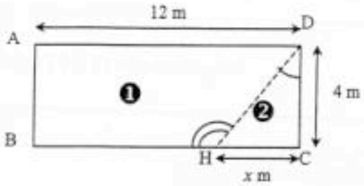
Exercice 3 (4 points)

Soit ABC un triangle tel que AB = 4,2 cm, BC = 5,6 cm, AC = 7 cm.
1) Faire une figure en vraie grandeur
2) Prouver que ABC est rectangle en B.
3) Calculer le périmètre et l'aire de ABC.

1.3. Part III : Problem

PROBLÈME (12 points)

On dispose d'un séjour rectangulaire dans lequel on veut réaliser un petit cagibi triangulaire. Pour cela on veut installer une cloison.



Voici ci-contre une représentation de la pièce.
La partie ❷ est le cagibi et la partie ❶ représente le séjour après la création du cagibi. La cloison a été dessinée en pointillés.

Dans l'exercice, on considèrera que la cloison a une épaisseur nulle.

Les trois parties sont indépendantes.

Partie I (3 points)

On considère ici que $x = 3$ m.

1) Quelle est la longueur de la cloison (en pointillé)?
2) Calculer la valeur (à 1° près) de l'angle $\widehat{HDC}$?
3) Calculer la valeur (à 1° près) de l'angle $\widehat{DHB}$?

Partie II : (6 points)

1) a) Exprimer la surface au sol du cagibi ❷ en fonction de x, sous la forme $f(x) = ...$
   b) Exprimer la surface au sol du séjour ❶ en fonction de x, sous la forme $g(x) = ...$

2) On admet que $f(x) = 2x$ et que $g(x) = 48 - 2x$.

   a) Quelle est la nature de la fonction f ? Quelle est la nature de la fonction g ?
   b) Tracer dans un repère (abscisse : 1 cm pour 0,5 unités et en ordonnées 1 cm pour 5 unités) les représentations graphiques des fonctions f et g pour x compris entre 0 et 10.

3) On veut que le séjour ❶ ait une surface minimale de 35 m².

   a) Lire sur le graphique la valeur maximale de x pour que cette condition soit respectée.
   b) Ecrire une inéquation qui traduise que la surface du séjour doit être supérieure où égale à 35 m².
   c) Résoudre cette inéquation.

Partie III (3 points)

On réalise une maquette de cette pièce, avant la création du cagibi, à l'échelle 1/200

1) Rappeler ce que signifie "échelle 1/200".
2) Quelle sera, sur la maquette, la longueur du mur de 12 m ?
3) La surface réelle du séjour est de 48 m². Quelle est la surface du sol du séjour dans la maquette (en cm²) ?
4) Le volume du séjour de la maquette est 13,125 cm³. Quel est le volume réel du séjour (en cm³ puis en m³).

Annexe 6 : Comparing PISA with the French curriculum

FRENCH MATHS SYLLABUS in LOWER SECONDARY SCHOOLS: SYNOPTIC CHART
Underlined bold : topics adressed by some PISA mathematical questions

	Classe de Sixième (GR 6)	Classe de Cinquième (GR 7)	Classe de Quatrième (GR 8)	Classe de Troisième (GR 9)
Configurations, constructions et transformations.	Cercle. Triangles, triangles particuliers. Rectangle, losange. Transformation de figures par symétrie axiale. **Parallélépipède rectangle.**	**Parallélogramme.** Construction de triangles (Instruments et/ou logiciel géométrique). Concours des médiatrices d'un triangle. Transformation de figures par symétrie centrale. Prismes droits, cylindres de révolution.	**Triangle : théorèmes relatifs aux milieux de deux côtés.** Triangles déterminés par deux droites parallèles coupant deux sécantes : proportionnalité de longueurs. Droites remarquables d'un triangle, leur concours. Triangle rectangle et son cercle circonscrit. Transformation de figures par translation. **Pyramides**, cône de révolution.	Polygones réguliers. Théorème de Thalès et réciproque. Transformation de figures par rotation ; composition de symétries centrales ou de translations. Vecteurs, somme de deux vecteurs. Sphère. Problèmes de sections planes de solides.
Repérage, distances et angles.	Abscisses positives sur une droite graduée. Repérage par les entiers relatifs, sur une droite graduée (abscisse) et dans le plan (coordonnées).	Repérage sur une droite graduée, distance de deux points. Repérage dans le plan (coordonnées). Inégalité triangulaire.	Relation de proportionnalité : représentation graphique. Théorème de Pythagore et sa réciproque. Distance d'un point à une droite. Tangente à un cercle. Cosinus d'un angle aigu.	Représentation graphique d'une fonction linéaire ou affine. Coordonnées du milieu d'un segment. Coordonnées d'un vecteur. Distance de deux points. Trigonométrie dans le triangle rectangle.
Grandeurs et mesures.	**Périmètre et aire d'un rectangle, aire** d'un triangle rectangle. Longueur d'un cercle. Volume d'un parallélépipède rectangle à partir d'un pavage .	Somme des angles d'un triangle. Aire du parallélogramme, du triangle, du disque. **Mesure du temps.** Aire latérale et volume d'un prisme droit, d'un cylindre de révolution.	**Grandeurs quotients courantes.** Volume d'une pyramide, volume et aire latérale d'un cône de révolution.	Grandeurs composées. Aire de la sphère, volume de la boule.
Nombres et calcul numérique.	**Écriture décimale et opérations +, -, x. Division par un entier** : quotient et reste dans la division euclidienne, **division approchée.** Troncature et arrondi. Écriture fractionnaire du quotient de deux entiers, simplifications.	Successions de calculs, priorités opératoires. Produit de fractions. Comparaison, somme et différence de fractions de dénominateurs égaux ou multiples. Comparaison, somme et différence de nombres relatifs en écriture décimale.	Opérations (+, -, x, :) sur les nombres relatifs en écriture décimale ou fractionnaire (non nécessairement simplifiée). Puissances d'exposant entier relatif. Notation scientifique des nombres. Touches √ et cos d'une calculatrice ; Inverses.	Calculs comportant des radicaux. Fractions irréductibles. Exemples simples d'algorithmes et applications numériques sur ordinateur.
Calcul littéral.	**Substitution de valeurs numériques à des lettres dans une formule.**	Égalités k(a+b) = ka + kb et k(a-b) = ka – kb. Test d'une égalité ou d'une inégalité par substitution de valeurs numériques à une ou plusieurs variables.	Développement d'expressions. Effet de l'addition et de la multiplication sur l'ordre. **Équations du premier degré à une inconnue.**	Factorisation (identités). **Problèmes se ramenant au premier degré.** Inéquations. Systèmes de deux équations du premier degré à deux inconnues.
Fonctions numériques.	**Application d'un taux de pourcentage.** Changements d'unités de longueur, d'aire. **Étude d'exemples relevant ou non de la proportionnalité.**	Mouvement uniforme. Calcul d'un pourcentage, d'une fréquence. **Changements d'unités de temps** et de volume. Coefficient de proportionnalité.	Vitesse moyenne. Calculs faisant intervenir des pourcentages. **Changements d'unités pour des grandeurs quotients courantes.** Applications de la proportionnalité.	Étude générale de l'effet d'une réduction, d'un agrandissement sur des aires, des volumes. Problèmes de changements d'unités pour des grandeurs composées. Fonctions linéaires et affines.
Représentation et organisation de données.	**Exemples conduisant à lire, à établir des tableaux, des graphiques.**	Classes, effectifs d'une distribution statistique. Fréquences. **Diagrammes à barres, diagrammes circulaires.**	Effectifs cumulés. Fréquences cumulées. **Moyennes.** Initiation à l'usage de tableurs-grapheurs.	Approche de la comparaison de séries statistiques.

Annexe 7 : Comparing a customary French examination with the French curriculum

FRENCH MATHS SYLLABUS in LOWER SECONDARY SCHOOLS: SYNOPTIC CHART
Underlined bold : topics adressed a customary final examination at grade 9 (Brevet 2005– South of France)

	Classe de Sixième (GR 6)	Classe de Cinquième (GR 7)	Classe de Quatrième (GR 8)	Classe de Troisième (GR 9)
Configurations, constructions et transformations.	Cercle. **Triangles, triangles particuliers.** **Rectangle**, losange. Transformation de figures par symétrie axiale. **Parallélépipède rectangle.**	Parallélogramme. **Construction de triangles (Instruments et/ou logiciel géométrique).** Concours des médiatrices d'un triangle. Transformation de figures par symétrie centrale. Prismes droits, cylindres de révolution.	**Triangle : théorèmes relatifs aux milieux de deux côtés. Triangles déterminés par deux droites parallèles coupant deux sécantes : proportionnalité de longueurs**. Droites remarquables d'un triangle, leur concours. Triangle rectangle et son cercle circonscrit. Transformation de figures par translation. Pyramides, cône de révolution.	Polygones réguliers. **Théorème de Thalès et réciproque.** Transformation de figures par rotation ; composition de symétries centrales ou de translations. Vecteurs, somme de deux vecteurs. Sphère. Problèmes de sections planes de solides.
Repérage, distances et angles.	Abscisses positives sur une droite graduée. Repérage par les entiers relatifs, sur une droite graduée (abscisse) et dans le plan (coordonnées).	Repérage sur une droite graduée, distance de deux points. Repérage dans le plan (coordonnées). Inégalité triangulaire.	Relation de proportionnalité : représentation graphique. **Théorème de Pythagore et sa réciproque**. Distance d'un point à une droite. Tangente à un cercle. **Cosinus d'un angle aigu.**	**Représentation graphique d'une fonction linéaire ou affine.** Coordonnées du milieu d'un segment. Coordonnées d'un vecteur. Distance de deux points. **Trigonométrie dans le triangle rectangle.**
Grandeurs et mesures.	**Périmètre et aire d'un rectangle, aire d'un triangle rectangle**. Longueur d'un cercle. **Volume d'un parallélépipède rectangle** à partir d'un pavage.	**Somme des angles d'un triangle**. Aire du parallélogramme, du triangle, du disque. **Mesure du temps.** Aire latérale et volume d'un prisme droit, d'un cylindre de révolution.	Grandeurs quotients courantes. Volume d'une pyramide, volume et aire latérale d'un cône de révolution.	Grandeurs composées. Aire de la sphère, volume de la boule.
Nombres et calcul numérique.	**Écriture décimale et opérations +, -, x.** Division par un entier : quotient et reste dans la division euclidienne, division approchée. Troncature et **arrondi**. **Écriture fractionnaire du quotient de deux entiers, simplifications.**	**Successions de calculs, priorités opératoires.** **Produit de fractions. Comparaison, somme et différence de fractions de dénominateurs égaux ou multiples**. Comparaison, somme et différence de nombres relatifs en écriture décimale.	Opérations (+, -, x, :) sur les nombres relatifs en écriture décimale ou fractionnaire (non nécessairement simplifiée). **Puissances d'exposant entier relatif. Notation scientifique des nombres.** **Touches √ et cos d'une calculatrice ; Inverses.**	**Calculs comportant des radicaux.** **Fractions irréductibles.** Exemples simples d'algorithmes et applications numériques sur ordinateur.
Calcul littéral.	**Substitution de valeurs numériques à des lettres dans une formule.**	Égalités k(a+b) = ka + kb et k(a-b) = ka – kb. Test d'une égalité ou d'une inégalité par substitution de valeurs numériques à une ou plusieurs variables.	**Développement d'expressions.** Effet de l'addition et de la multiplication sur l'ordre. **Équations du premier degré à une inconnue.**	Factorisation (identités). **Problèmes se ramenant au premier degré. Inéquations.** Systèmes de deux équations du premier degré à deux inconnues.
Fonctions numériques.	Application d'un taux de pourcentage. **Changements d'unités de longueur, d'aire.** Étude d'exemples relevant ou non de la proportionnalité.	Mouvement uniforme. Calcul d'un pourcentage, d'une fréquence. **Changements d'unités de temps et de volume.** Coefficient de proportionnalité.	Vitesse moyenne. Calculs faisant intervenir des pourcentages. Changements d'unités pour des grandeurs quotients courantes. Applications de la proportionnalité.	**Étude générale de l'effet d'une réduction, d'un agrandissement sur des aires, des volumes.** Problèmes de changements d'unités pour des grandeurs composées. **Fonctions linéaires et affines.**
Représentation et organisation de données.	**Exemples conduisant à lire, à établir des tableaux, des graphiques.**	Classes, **effectifs d'une distribution statistique. Fréquences.** **Diagrammes à barres**, diagrammes circulaires.	**Effectifs cumulés**. Fréquences cumulées. **Moyennes.** Initiation à l'usage de tableurs-grapheurs.	Approche de la comparaison de séries statistiques.